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ABSTRACT

This syllabus for grade 7 was designed to improve the problem-solving abilities of students while continuing a natural development of mathematical skills and concepts. Seven major strands are provided under the following unit titles: (1) "An Introduction to Problem Solving"; (2) "Number Concepts"; (3) "Ratio, Proportion, and Percent"; (4) "Probability and Statistics"; (5) "Algebra and Coordinate Geometry"; (6) "Computer Awareness"; and (7) "Geometry and Measurement." Following the statement of goals and objectives, a content outline and suggested time allotments are presented. Detailed suggestions to teachers are given for each strand, with background information, learning activities with materials and diagrams, and examples included for each topic on the content outline. Evaluation is considered briefly, and appendices present the Statement of Regents Goals (1984), comments on providing for students with handicapping conditions, and the content outline for grade 8. (MNS)

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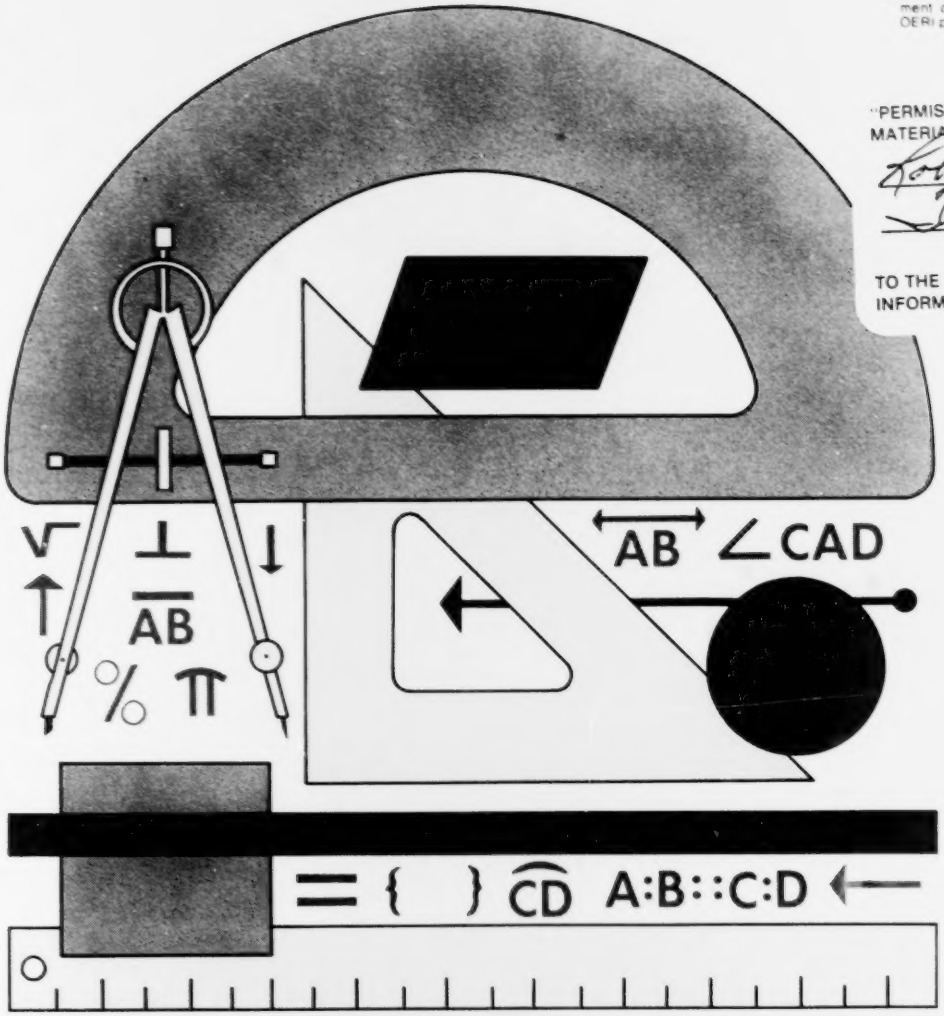

MATHEMATICS

7 SYLLABUS

NEW YORK
THE STATE OF LEARNING

The University of the State of New York / THE STATE EDUCATION DEPARTMENT
Bureau of Mathematics Education / Bureau of Curriculum Development
Albany, New York 12234
1986

SE 047 731

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TO: Persons with Responsibilities for Implementing Mathematics Programs

FROM: Edward T. Lalor, Director, Division for Program Development

Charles J. Trupia, Director, Division of General Education

This publication, *Mathematics 7*, is the syllabus designed to continue the developmental mathematics program established by *Mathematics K-6: A Recommended Program for Elementary Schools*. It is intended to serve as the basis for local activities such as:

Development of local courses of study to meet the standards stated in the syllabus

Selection and acquisition of support materials

Articulation of local offerings from grade to grade or building to building

Evaluation of student performance

As is the policy of the State Education Department, this syllabus will be periodically reviewed. Your comments on this syllabus are welcome and should be addressed to:

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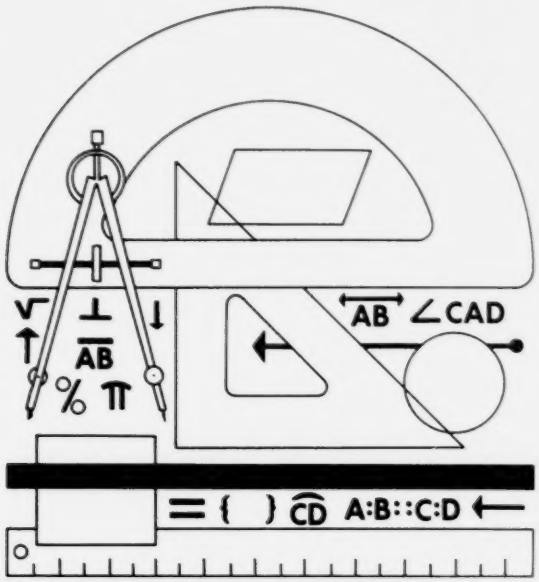

MATHEMATICS 7
Syllabus

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FOREWORD

This document has been prepared to help teachers and administrators plan and execute a mathematics program for grade 7.

In both content and approach, the units reflect the guidelines and philosophy formulated by an advisory committee of mathematics teachers assembled in the spring of 1981:

Rose Ambrosino, Smithtown
William Collins, Syracuse
Joyce Evans, Afton
Herbert Fremont, Queens College
Eileen Gress, Fairport
Marvin Itzkowitz, New York City
Norman Michaels, Rochester
Susanne Reeder, Buffalo

Two of the committee members, the late Herbert Fremont and Eileen Gress, were selected as writers. By January of 1982, a content outline and the beginnings of a first draft were ready for review. Teachers then engaged in teaching mathematics in grade 7 were asked to read the material and submit their comments in writing. These were compiled during the summer of 1982. In November 1982 a new writer, Edward Keenan of the East Williston Schools, was commissioned to write a *Syllabus* for mathematics in grade 7 that:

- Was based on the guidelines and philosophy expressed by the advisory committee,
- Incorporated aspects of the material already developed and the suggestions of the reviewers, and
- Was suitable for fieldtesting.

In the spring of 1983, 41 pilot districts were selected from over 100 volunteers. Care was taken to choose schools that reflected all types of districts (urban, rural, and suburban) in all regions of the State. The districts included:

Attica
Auburn
Ballston Spa
Bay Shore
Binghamton (parochial)
Buffalo
Burnt Hills
Carthage
Cheektowaga
Clinton
Cobleskill
Division for Youth
Dobbs Ferry
Duanesburg
East Greenbush
East Syracuse
Fairport
Geneva
Hicksville
Iroquois
Kenmore
Madrid-Waddington
Manhasset
Mastic Beach
Miller Place
Mt. Vernon
New Hartford
Newburgh
New York City
Norwood-Norfolk

Orchard Park
Pelham
Riverhead
Schenectady
Seneca Falls
Utica
Vestal
Wappingers Falls
Watervliet
West Irondequoit
Whitesboro

In June 1985, after two years of pilot testing, these schools evaluated the document and sent their comments to the Bureau of Mathematics Education. These evaluations were compiled and analyzed by Norman Swanson of Hudson Valley Community College. During July and August 1985 Joseph Gehringer of the New York City School District #2 and Edward Keenan revised the draft material to reflect the concerns expressed in the evaluations.

This *Syllabus* for mathematics in grade 7 was prepared under the direction of Dorothy Foley, Chief, Bureau of Curriculum Development, and Fredric Paul, Chief, Bureau of Mathematics Education. Lynn A. Richbart and Toni Meyer, Associates, Bureau of Mathematics Education, coordinated the field testing and evaluation of the manuscript. Staff of the Office of Education of Children with Handicapping Conditions reviewed the manuscript in terms of the needs of students with handicapping conditions. Michael Moon, Associate, Division of Civil Rights and Intercultural Relations, reviewed the manuscript for equity concerns. Rita Sator and Robert Carruthers, Associates, Bureau of Curriculum Development, prepared its format and edited the manuscript.

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INTRODUCTION

A. *Content of the Syllabus*

This *Syllabus* for grade 7 is designed to improve the problem-solving abilities of students, while continuing a natural development in areas such as mathematical skills and concepts. To foster this approach, problem-solving situations are an integral part of each strand of study.

In extending the pattern established in the K-6 *Syllabus*, the *Syllabus* for grade 7 is organized into seven major strands of study:

I. An Introduction to Problem Solving
II. Number Concepts
III. Ratio, Proportion, and Percent
IV. Probability and Statistics
V. Algebra and Coordinate Geometry
VI. Computer Awareness
VII. Geometry and Measurement

B. *Instructional Considerations*

Although strands are listed separately, it is expected that there will be many points of intersection between them resulting in a unified program in mathematics for grade 7, through the development of activities that grow out of the following fundamental principles:

- Problem solving is the basic approach to the program and is utilized in every strand.
- After studying basic operations with whole numbers, common fractions, and decimals in the Number Concepts strand, students should apply these skills as an integral part of problem-solving activities in each strand.
- Integration of topics should occur naturally and not be forced.
- Mathematics is to be explored at an informal level; wherever possible, formal operations and concepts are to be developed through the use of concrete models.
- Alternative algorithms are presented, and their use is encouraged.
- Although students still must demonstrate competence in mathematical operations, computers and hand-held calculators are valuable tools that may be used in developing concepts, in finding patterns, in checking work, and (as in statistics) in removing some of the computational drudgery from mathematical problem solving. The use of computers and hand-held calculators may be especially important for students who have the ability to acquire the concepts and skills presented in this *Syllabus* but who, because of a handicapping condition, have difficulty in demonstrating competence in computation.

C. *Use of the Syllabus*

It is recommended that the study of mathematics in grade 7 begin with problem solving. The order in which the other strands are presented is not intended to indicate the order in which the

topics must be studied. Teachers should feel free to develop these mathematical ideas in a sequence that seems best to them.

In the material which follows, several topics are optional, and should be included only as time and student ability permit.

A suggested time allotment is given for each major unit and is intended as a general guide in planning the entire year's work.

Suggested instructional strategies are included for each topic within the outline. These strategies are intended to give teachers ideas, methodologies, and examples of what might be done within a particular topic. Teachers are encouraged to adapt these strategies to their own particular style of teaching and to the individual needs of students.

GOALS AND OBJECTIVES

The Regents Statement of Goals for students includes the following:

1. Each student will master computation skills as a foundation to
   - think logically and creatively
   - apply reasoning skills to issues and problems
   - perform basic mathematical calculations
   - determine what information is needed for particular purposes and be able to acquire, organize, and use that information for those purposes.
2. Each student will learn methods of inquiry and knowledge through mathematics and use the methods and knowledge in interdisciplinary applications.

This *Syllabus* uses these Regents Goals as a cornerstone. Its objectives support the National Council of Teachers of Mathematics' *An Agenda for Action*, which recommends that problem solving be the focus of school mathematics and that basic skills in mathematics be defined to encompass more than computational facility.

The National Council of Supervisors of Mathematics' *Basic Mathematical Skills* defines ten such areas.

1. Problem Solving
2. Applying Mathematics to Everyday Situations
3. Alertness to the Reasonableness of Results
4. Estimation and Approximation
5. Appropriate Computational Skills
6. Geometry
7. Measurement
8. Reading, Interpreting, and Constructing Tables, Charts, and Graphs
9. Using Mathematics to Predict
10. Computer Literacy

Specific content objectives relating to each of these basic skills are listed in the Content Outline of this *Syllabus* and are described in its Suggestions to Teachers.

CONTENT OUTLINE AND SUGGESTED TIME ALLOTMENTS

Unit Topics	Time (Days)
I. An Introduction to Problem Solving	4–5

A. General Technique

1. Understand what is happening in the problem
2. Formulate a plan for solution
3. Carry out the plan and observe the consequences
4. Check the answer to see whether it makes sense

B. Various Problem-Solving Strategies

1. Use of pictures and diagrams
2. Use of a simpler but related problem
3. Patterns
4. Trial and error (guessing)
5. Working backwards
6. Insight
7. Miscellaneous examples

II. Number Concepts — 45–49

A. Whole Numbers

1. Definition of natural numbers or counting numbers
2. Definition of whole numbers
   a. Reading and writing numerals
   b. Rounding whole numbers
3. Operations with whole numbers
   a. Properties
   b. Inverse operations
4. Sequences
5. Factors and exponents
6. Prime and composite numbers
7. Multiples and divisors
   a. Greatest common factor
   b. Least common multiple
   c. Rules for divisibility
8. Principal square roots that are whole numbers
9. Order of operations

B. Integers

1. Definition
2. Absolute value
3. Addition of integers
   a. Subtraction of integers
   b. Problem solving

4. Multiplication of integers
    a. Division of integers
    b. Problem solving
5. Order of operations

C. Rational numbers

1. Definition and subsets of the rational numbers
2. Equivalent fractions
    a. Reducing to lowest terms
    b. Increasing to higher terms
3. Operations and problem solving with positive common fractions
    a. Four basic operations
    b. Simplifying complex fractions
    c. Order of operations
4. Converting common fractions to decimal fractions
    a. Terminating decimals
    b. Repeating decimals
5. Converting decimal fractions to common fractions
6. Operations and problem solving with decimals
    a. Four basic operations
    b. Estimation
    c. Rounding decimals
7. Scientific notation for numbers larger than or equal to one
8. Inequalities
    a. Comparisons of rational numbers
    b. Number line graphs
    c. Problem solving

III. Ratio, Proportion, and Percent 25–30

A. Ratio

1. Definition and forms of notation
2. Rates as ratios
3. Problem solving

B. Proportion

1. Definition and forms of notation
    a. Cross-products rule
    b. Means and extremes
2. Solving proportions
3. Problem solving

C. Percent

1. Definition
2. Decimal/fraction/percent equivalents
3. Working with percents
    a. Finding a percent of a number
    b. Finding what percent one number is of another
    c. Finding a number when a percent of it is given
4. Problem solving

IV. Probability and Statistics 23–26

A. Probability

1. Empirical vs. theoretical probability
2. Empirical probability
   a. Cumulative relative frequency
   b. Biased objects
3. Theoretical probability
   a. Sample space or outcome set
   b. Event
   c. Random selection
4. Range of probabilities
   a. Certainty
   b. Impossibility
   c. Probabilities between zero and one
   d. Sum of the probabilities
   e. Probability of complement
5. Probability of two independent events
   a. Definition of independent events
   b. The counting principle (multiplication principle)
   c. Sample spaces for two independent events
      (1) Tree diagrams
      (2) List of ordered pairs
      (3) Graph of ordered pairs
   d. Product of independent probabilities

B. Statistics

1. Definition
   a. Steps in a statistical study
   b. Sample
2. Organization of data
   a. Range
   b. Intervals of equal length
   c. Frequency distribution
3. Histogram
4. Measures of central tendency
   a. Mean
   b. Median
   c. Mode
   d. Problem solving

V. Algebra and Coordinate Geometry 20–23

A. Algebraic Expressions and Sentences

1. Concepts through applications
   a. Variables, constants and domains
   b. Open sentence vs. algebraic expression
2. Translating verbal phrases into algebraic form
   a. Forming algebraic expressions
   b. Evaluating algebraic expressions

B. First Degree Equations and Inequalities in One Unknown

1. Forming open sentences
2. Solving open sentences that are equations
   a. Replacement
   b. Inverse operations
3. Solving open sentences that are inequalities

C. Functions and the Coordinate Plane

1. Concepts through applications (functions of the form $y = ax$)
   a. Table of values
   b. Sentence
   c. Formula
   d. Coordinate graph (first quadrant)
   e. Predictions
2. Functions of the form $y = ax + b$ (*optional*)
3. The coordinate plane

VI. Computer Awareness 8–14

A. Introduction to Computers

1. What is a computer?
2. Components of a modern-day digital computer
   a. Input
   b. Control
   c. Arithmetic and logic unit
   d. Memory
   e. Output
3. Organizing a plan for a computer solution
   a. Flowchart
   b. Program
4. Uses of computers today
   a. School
   b. Home
   c. Business/Industry
   d. Government
   e. Media

B. Mathematical Operations and Functions

1. Operations using computer symbols
2. Evaluating expressions in computer format
   a. Order of operations
   b. Use of decimals in output
   c. Use of scientific notation in output
3. Functions used with computers
   a. Absolute value function, ABS(X)
   b. Greatest integer function, INT(X)
4. Capabilities and limitations

C. Computer Programming (*Optional*)

1. Determine computer output of given programs
2. Algorithms used in programming
3. Write original programs
4. Additional activities

VII. Geometry and Measurement 20–23

A. Geometry

1. Undefined terms: point, line, plane
2. Line segment, ray, and pairs of lines
3. Angles
   a. Definition and symbolism
   b. Degree measure and classification
   c. Protractor
   d. Pairs of angles: complementary, supplementary, adjacent, and vertical
4. Simple closed curves
5. Polygons
   a. Definition and types of polygons
   b. Triangles
      (1) Classification by angles
      (2) Classification by sides
   c. Quadrilaterals
      (1) Trapezoid
      (2) Parallelogram
      (3) Rectangle
      (4) Rhombus
      (5) Square
   d. Problem solving
6. Circles

B. Metric System

1. Basic units of length, mass, and capacity
2. Prefixes and symbols
3. Equivalent measures within the metric system
4. Other measures: temperature, time, area, and volume

C. Customary System of Measure

D. Measurement of Geometric Figures

1. Perimeter
   a. Polygons
   b. Circles: circumference
2. Area
   a. Quadrilaterals
      (1) Rectangle
      (2) Square
      (3) Parallelogram
      (4) Rhombus
      (5) Trapezoid (*optional*)
   b. Triangles
   c. Circles
3. Volume
   a. Rectangular prism
   b. Cube

SUGGESTIONS TO TEACHERS

I. AN INTRODUCTION TO PROBLEM SOLVING

It is strongly recommended that an introduction to problem solving be taught as the first unit in grade 7, since an increased ability to solve problems is a goal to be infused within all other strands of this course.

This introduction should provide the students with a general technique for solving problems. Then, by presenting problems from several of the curriculum strands, various problem-solving strategies can be discussed. These strategies should serve as a catalogue of approaches to be known by the students so that they may choose to attack problems in different ways. In other words, while the students should be taught different strategies, they should also be encouraged to solve problems on their own.

In later years, phrases such as "only an algebraic solution will be accepted" do place limits on the approaches to be used with certain problems. In grade 7, however, a student should not be required to use a specific strategy to solve a specific problem.

Problems, both standard and nonstandard in nature, should be presented in this introduction and throughout the course. The solutions to some problems need not be hurried; a problem may be discussed over a period of days while the students attempt to find its solution by various means. The discussion of a misguided approach suggested by a student may sometimes uncover clues that lead to a correct solution.

The students should be aware that our developing and complex world continues to present new situations that require creative problem-solving techniques. Today's students will probably be required as adults to solve problems that our present society cannot even begin to describe.

A. GENERAL TECHNIQUE

At its simplest level, there are four steps to be taken in solving any problem:

1. Understand what is happening in the problem.

The problem must be read and reread. Important data and clues that lead to a solution should be noted, while extraneous information should be discarded.

2. Formulate a plan for solution.

Approaches to be used, and mathematical operations to be employed, should be identified. If the plan requires many steps, know the order of the procedures to be used.

3. Carry out the plan and observe the consequences.

Here, various problem-solving strategies are used. Care should be taken in organizing the work to be done in reaching a solution because it may be necessary to reexamine this work at a later time.

4. Check the answer to see if it makes sense.

The problem should be read again to make certain that the question has been answered properly. See that the answer falls within the bounds of common sense, whether using estimation or reasoning skills.

These four steps may be learned by the use of an abbreviated schema, such as "Understand-Plan-Solve-Check," or "Read-Organize-Solve-Interpret," which can serve as a point of reference for problem solving throughout the course.

The teacher should realize that the techniques outlined here, although sound and correct, represent an approach to problem solving in its most basic form. On a higher level, the teacher should at times alter the constraints and data in a problem after it has been solved. Excitement, as well as a deeper understanding of the problem, may be generated by asking some conditional questions: "If the length of the taxi ride is doubled, would the fare be doubled?," or "We found the fare for each of three people who shared a taxi. What should each person be charged if one of them got out about midway during the ride?"

There are a number of good references in the field of problem solving that are available to the teacher, notably the works of Polya; an assortment of materials from the National Council of Teachers of Mathematics; and, a booklet entitled *Creative Problem Solving*, published by the New York State Education Department.

B. VARIOUS PROBLEM-SOLVING STRATEGIES

Each of the problems that follow is given here to illustrate a specific strategy in finding the solution. However, the students may suggest alternate approaches in

the classroom. Encourage this type of thinking by following up on as many suggestions as possible, whether correct or incorrect.

In some cases, the students will see that various strategies may be used to solve the problem. In other cases, a student suggestion may lead to a "dead end." There is still much to be learned by trying an incorrect strategy, including a discussion of why the solution could not be reached with this approach as well as the chance that it may lead to an alternate strategy and the correct solution.

1. Use of Pictures and Diagrams

Example: I wish to give a dinner party for 22 people, including myself. I am going to borrow square card tables, the kind that seat one person on a side. If I arrange the tables in a long row, end to end, what is the fewest number of tables I should borrow?

Follow the steps for the general technique to be used in problem solving by asking questions.

Read: "Do you understand what is happening in the problem?" Have the students explain the situation in their own words by indicating what information is provided, what special conditions are imposed, and what they are being asked to find.

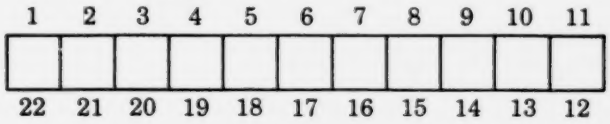
Plan: "Can you suggest a way to solve this problem?" Some students may wish to draw a picture; some may suggest the use of a chart to develop a pattern; others may offer different approaches. Encourage student verbalization, and accept the various plans without criticism.

Solve: "Can we carry out your plan and observe the consequences?" Student solutions rather than teacher demonstrations should be stressed. If necessary, the teacher may ask questions or provide some clues as a starting point.

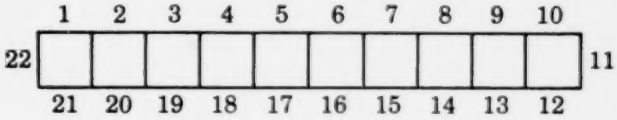
"How do we start to draw the tables?"

"Write 1 through 22 to show the people."

It is not uncommon that answers will vary in a classroom. A student might claim that, "eleven tables are needed," using the following diagram:



Check: "Can we check the answer to see if it makes sense?" The problem should be read again. What did the problem ask us to find? Did we find it? Since we were asked to find the *fewest* number of tables, and since people could be seated at the extreme ends, we need one less table:



Answer: 10 tables

Alternative Approach: Ask the students if someone found another approach. For example, a student may have used patterns as well as pictures in reading the solution.

	Number of Tables	Number of People
	1	4
	2	6
	3	8
	10	

Each new table placed in the row adds	4	10
2 more seats. Keep following the	5	12
pattern until 22 people are seated.	6	14
	7	16
	8	18
	9	20
Answer: 10 tables are needed.	10	22

---

2. Use of a Simpler but Related Problem

Example: The school hallway has a floor of tiles in a pattern of 4 white tiles and 1 red tile. If 1000 tiles are needed, how many should be white and how many red?

The steps outlined in the general technique, that is "Read-Plan-Solve-Check," are again reenforced through questioning which is left to the teacher and not included hereafter in this *Syllabus*.

Start with an easy example.

5 tiles:

R	W	W	W	W

1 red, 4 white

Try another easy example. (This configuration is used for the examples that follow.)

10 tiles:

R	W	W	W	W
R	W	W	W	W

2 red, 8 white

So, for 100 tiles, it is 10 times as many. 100 tiles: 20 red, 80 white

And for 1000 tiles, it is 100 times as many. 1000 tiles: 200 red, 800 white

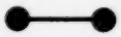
The easier but related examples show us how to solve the problem.

Answer: 200 red tiles and 800 white tiles.

3. Patterns

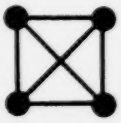
Example: There are 6 people at a party. If each person shakes hands once and only once with every other person, how many handshakes are there altogether?

Pictures, as well as the use of simpler but related problems, help to present the pattern. In each picture, let a point represent a person and let a segment connecting two points represent a handshake.



2 people — 1 handshake

3 people — 3 handshakes

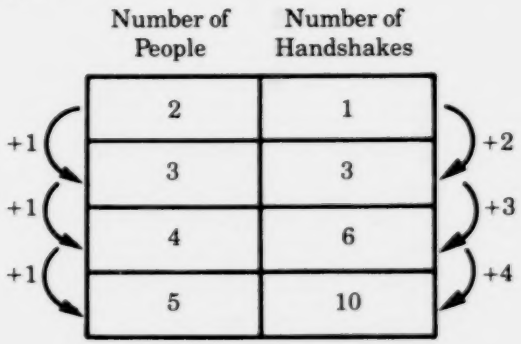


4 people — 6 handshakes

5 people — 10 handshakes

Stop! Before drawing a picture to represent 6 people, study the table to discover a pattern.

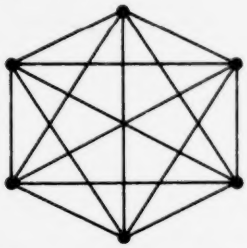
The number of people keeps increasing by 1. But the number of handshakes keeps increasing by a different amount: first by 2, then by 3, then by 4. Can you find the pattern?



Answer: 15 handshakes

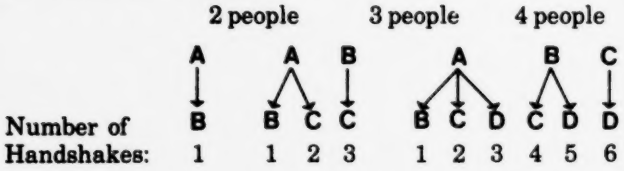
If the number of people increases from 5 to 6, how many handshakes are there altogether? There will be 5 more than 10, for a total of 15 handshakes.

To test your answer, draw a picture with 6 points representing 6 people and find the 15 segments that represent the 15 handshakes.



Without drawing pictures, have the students tell how many handshakes there would be if the party was attended by 7 people, then 8, 9, 10, 11, and 12 people.

Alternative Approach: A student may use letters such as A, B, C, . . . to represent people such as Ann, Bill, Carol, . . . , and so forth. Other pictures could lead to the patterns. For example:



(The pattern may also be developed by having the students act out the actual handshakes at a party for 2, 3, 4, and 5 people.)

4. Trial and Error (Guessing)

Example: Toni and Lynn know their heights in metric measures. If the sum of their heights is 314 cm, and the difference of their heights is 10 cm, find both heights.

Guess any two numbers whose sum is 314; then find their difference.	214 +100 314	214 −100 114 (too large)
Subtract some amount, say 40, from the larger number guessed and add the same amount, 40, to the smaller number guessed. The sum is still 314. Again, find the difference.	174 +140 314	174 −140 34 (closer; still too large)
Continue the process. Here, 14 is subtracted from 174, and 14 is added to 140.	160 +154 314	160 −154 6 (now too small)
Add 2 to 160, and subtract 2 from 154 to keep the sum at 314. Find the difference.	162 +152 314 (true)	162 −152 10 (true)

Answer: The heights are 162 cm and 152 cm.

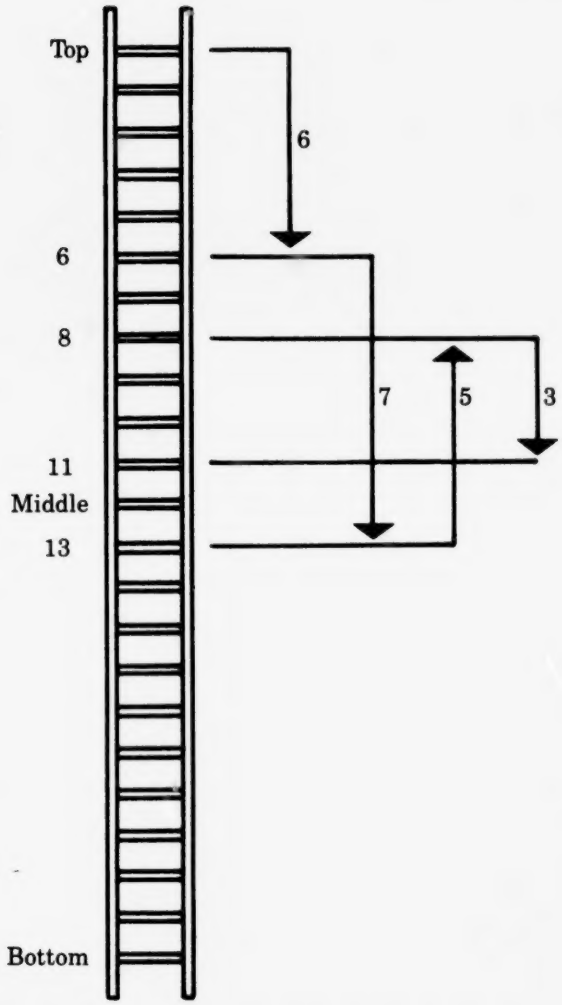
Discuss the strategy and the problem. If we had started by guessing two other numbers, would this strategy eventually get us to the correct solution? Who is taller, Toni or Lynn? Why can we not tell? If the question read, "Toni is 10 cm taller than Lynn," could we then determine each person's height?

5. Working Backwards

Example: A fireman spraying water on a fire stood on the middle rung of a ladder. The smoke lessened, so he moved up 3 rungs. It got too hot, so he backed down 5 rungs. Later he went up 7 rungs and stayed until the fire was out. Then he climbed the 6 remaining rungs and went into the building. How many rungs does the ladder have?

Work backwards to solve the problem. At the end, the fireman went up 6 rungs to get to the top. So he started from the 7th rung below the top. Before that, he climbed up 7 rungs; if we move down 7, we are on the 14th rung from the top. Continue working backwards in this fashion until the middle rung is reached. This is the 12th rung from the top. Therefore, we know.

11 rungs above the middle
11 rungs below the middle
\+ 1 middle rung
23



Answer: 23 rungs total on the ladder

Check the problem by starting on the middle rung of the ladder in the picture drawn. Read the problem and make each appropriate move until the fireman enters the building.

6. Insight

Continue problem solving, stressing the strategies described above. It is critical that the students be given the freedom and the encouragement to solve problems in their own way. They should be taught various strategies as indicated above. However, THEY SHOULD *NOT* BE REQUIRED TO SOLVE PROBLEMS USING A PARTICULAR STRATEGY. In this way they will have a catalogue of approaches to use, and it will be the students' decision as to when to use a particular strategy and in what context. The students must be faced with problems in which the approach is not apparent, and encouraged to devise and test alternative approaches. They must learn to think for themselves.

At times a student might gain a flash of INSIGHT, and see an immediate solution to a problem. Although the student may lack a complete explanation for the solution, encourage the student to verbalize the clues that led to that conclusion. Creative thinking and the "Eureka" effect should be nurtured, not rejected. A discussion of the clues, no matter how limited, can often help others in the class to see the strategy being used or to see an alternate approach.

Remember to let the students experience problems where they must think about a solution over many days. Tell the students about "classic" problems that mathematicians have been unable to solve for centuries. Another classic problem, namely the 4 color problem in maps, went unsolved for years but was recently proved using a computer. New ways of thinking, new approaches, and insight will all play important roles in problem solving in the future.

7. Miscellaneous Examples

The teacher may include some or all of the problems that follow to supplement this unit, or may present a collection of favorite problems for student analysis and classroom discussion.

Example: If the sum of two numbers is 43 and the product of these numbers is 420, find the numbers.

Answer: 28 and 15

Example: If 5 students can assemble 5 toys in 5 minutes, how long will it take 10 students to assemble 10 toys?

Answer: 5 minutes

Example: A centipede is in a well, exactly 1 meter from the top. The centipede climbs up 16 cm in one minute, then rests and slips back down 2 cm on the slick wall in the next minute. If the centipede keeps gaining 16 cm and losing 2 cm in alternate minutes, how long will it take it to get out of the well?

Answer: 13 minutes

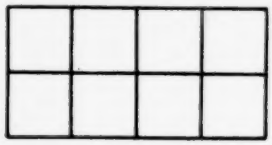
Example: A rectangular piece of paper contains 15 squares arranged in 3 rows of 5 each. How many L-shaped pieces consisting of exactly 3 squares can be cut out of the given paper?

Answer: 4

Example: I am thinking of a counting number less than 23. The number is not a divisor of 60. The number is not odd, but it is a multiple of 3. What is the number?

Answer: 18

Example: How many different rectangles can be found in the accompanying figure? Remember, every square is also a rectangle.



Answer: 30

Example: Granny Gillen loves pets, but she keeps only birds and dogs in her pet store. If her pets amount to "40 mouths to feed," and they have a total of 94 legs, how many of these pets are birds?

Answer: 33

II. NUMBER CONCEPTS

In this unit, student weaknesses in computational skills with whole numbers, fractions, and decimals can be addressed and remedied. The aim is for all students to become proficient in adding, subtracting, multiplying, and dividing whole numbers, positive common fractions, and positive decimal fractions.

The problem-solving techniques developed throughout the introductory unit should be applied to this work with number concepts. Difficulties that the students may have with basic computational skills may be considered as they arise from the work included here. In this way, the development of computational skill will be seen by the student as a means to a desirable end, rather than an end in itself.

Although integers were introduced in grades 5 and 6, the present unit will include the first formal study of operations with integers.

Some teachers may wish to integrate Unit VI, Computer Awareness, within this study of number concepts to set the stage for computer usage throughout the *Syllabus*. Other teachers may delay the computer unit until later in the year, to reenforce number concepts in a spiral learning approach.

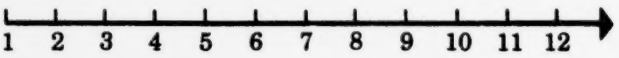
A. WHOLE NUMBERS

1. Definition of Natural Numbers, or Counting Numbers

The *natural numbers*, which are also called the *counting numbers*, are 1, 2, 3, 4, . . . The smallest natural number is 1. Every natural number has a *successor* that is one more than the given number. The successor of 1 is 2; the successor of 2 is 3. In general, the successor of the natural number n is $n + 1$. There is no largest natural number.

Set notation, *language* and *operations*, which appear in the K-6 *Syllabus*, will be used throughout this curriculum. Thus, $\{1, 2, 3, 4, \ldots\}$ indicates the set of natural numbers. The set of natural numbers offers us an example of an *infinite set*, that is, one containing an endless number of elements, or one whose members cannot be counted. A *finite set*, or one whose members can be counted, is illustrated by $\{5, 11\}$ or $\{7\}$. The *empty set* or *null set*, indicated by $\{\ \}$ or Φ, contains no members.

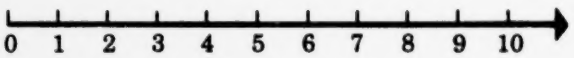
A *number line* (or any part thereof) represents each number geometrically by a point on the line. Since the difference between successive natural numbers is always one, the same distance is used between points representing these numbers on a line. A number line may be drawn in any direction, the most common being horizontal and vertical.



Natural Numbers or Counting Numbers

2. Definition of Whole Numbers

The set of *whole numbers* is defined to be the set containing the natural numbers and zero, namely $\{0, 1, 2, 3, 4, 5, \ldots.\}$ Note that zero is not a natural number, nor is it a counting number.



Whole Numbers

a. Reading and writing numerals

In grades 5 and 6, students were to read and write numerals for whole numbers up to billions. The teacher may indicate that groups of three digits have traditionally been separated by commas as an aid to reading the numerals. In the International System of Units (SI), spaces rather than commas are used to separate the groups of three digits. Both procedures are acceptable.

9 1 7,	0 8 5,	4 6 3,	2 7 8
billions	millions	thousands	ones
9 1 7	0 8 5	4 6 3	2 7 8

b. Rounding whole numbers

In grades 5 and 6, whole numbers were rounded to the nearest ten thousand. In grade 7, rounding skills should be extended to any place up through hundred billions.

Example: Round 917,085,463,278 to the nearest ten million.

Answer: 917,090,000,000

Example: Round 917 085 463 278 to the nearest billion.

Answer: 917 000 000 000

Example: Park officials estimated that 28,000 people were at the beach on Saturday. If the estimate given was correct to the nearest thousand, which number could *not* be a more accurate count of people at the beach?

a. 27,500 *b.* 28,495 *c.* 28,097 *d.* 27,380

Answer: d.

Optional: Larger numerals may be presented through a study of prefixes. Such numerals, however, are more often written in scientific notation, which will be studied later in this unit.

Numeral	Prefix	Meaning of Prefix
billion	bi-	2
trillion	tri-	3
quadrillion	quadr-	4
quintillion	quin-	5
sextillion	sex-	6
septillion	sept-	7
octillion	oct-	8
nonillion	non-	9
decillion	dec-	10

3. Operations with Whole Numbers

The operations of addition, subtraction, multiplication, and division with whole numbers should be studied through a series of problems, and the problem-solving techniques learned earlier should be applied to these situations. Some problems should contain multiple steps; others should contain extraneous information.

Example: Mr. Novak assigned the following problems for homework:

page 30, #1 through #8
page 31, #12 through #21
page 35, #3 through #17 (odd numbers only)

How many problems did Mr. Novak assign?

Answer: 26

Example: An average person's heart beats 74 times per minute; a well-trained athlete's heart beats 55 times per minute. Over a period of one hour, how many more times does an average person's heart beat than that of a well-trained athlete?

Answer: 1,140

Example: A supermarket has received 2,880 eggs shipped in 8 large crates. If there are 12 eggs to a carton, how many cartons fit into a crate?

Answer: 30

Example: A timing device turns on a lamp at 7:00 p.m. daily, and turns off the lamp 6 hours later. After 180 days, the bulb burns out. Find the total number of hours that the light bulb burned.

Answer: 1,080

(What information is extraneous to the problem stated above? If the problem only asked when the lamp is turned off, what information would then be extraneous?)

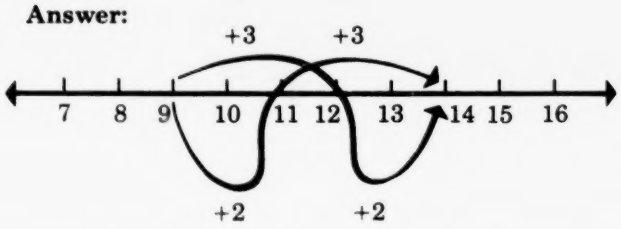
a. Properties

In K-6, properties applied to operations were studied from a numerical point of view. For example, $8 + 17 = 25$ and $17 + 8 = 25$ illustrate the commutative property of addition. The students in grade 7 need not know the names of these properties nor their symbolic form. The teacher, however, should correctly name each property as it occurs or is used in a mathematical situation.

(1) Commutative Property of Addition:
$a+b = b+a$
(2) Commutative Property of Multiplication:
$a \cdot b = b \cdot a$
(3) Associative Property of Addition:
$(a+b) + c = a + (b+c)$
(4) Associative Property of Multiplication:
$(a \cdot b) \cdot c = a \cdot (b \cdot c)$
(5) Distributive Property of Multiplication over Addition: $a \cdot (b+c) = a \cdot b + a \cdot c$

Example: Start at any point on the whole number line. Using arrows for +3 and +2, find two different paths each of which has a value of +5. That is, demonstrate that +3+(+2) = +2+(+3).

Answer:



Example: Find the missing value to make the sentence true:

$3 \times 58 = (3 \times 50) + (3 \times \quad)$

Answer: 8

Example: Palindromes (*optional*)

A study of palindromes provides both practice in the addition of whole numbers and another avenue to investigate number patterns.

A *palindrome* is a number (or word) that reads the same left to right and right to left. For example: 51315 is a palindrome, as are 8341661438 and the word RADAR.

Pick any 2-digit number. Reverse the digits and add. Is the sum a palindrome? If not, reverse it and add again. Repeat until you get a palindrome. For example:

Pick 86	86	
REVERSE IT	+68	
	154	no palindrome
	+451	
	605	no palindrome
	+506	
Answer:	1111	A PALINDROME!

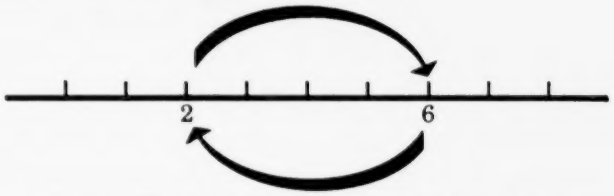
The students can be asked to classify sets of 2-digit numbers by the number of additions needed to create a palindrome. For example, 86 (as shown) is a class 3 number since 3 additions were needed to form a palindrome. While most 2-digit numbers require only one or two additions, some require extensive work to form a palindrome: 79 is a class 6 number, and 98 is a class 24 number.

This activity, extended to 3-digit numbers, might best be handled by using a calculator.

b. Inverse operations

In earlier grades, the students learned that addition and subtraction are inverse operations, as are the operations of multiplication and division. In grade 7, these concepts can be reenforced on a number line, and a formal treatment of inverses and identity elements given later in this unit.

Example: In the accompanying diagram, the arrows represent inverse operations. Give at least two names for each pair of arrows.



Answer: The top arrow is +4 or "plus 4," and the bottom arrow is –4 or "minus 4." Or, the top arrow is × 3 or "multiplied by 3," and the bottom arrow is ÷ 3 or "divided by 3."

4. Sequences

Critical thinking skills can be enhanced through a study of sequences and games such as "Guess My Rule."

Functions are used in the game, "Guess My Rule," so that for every number given, there is exactly one and only one answer. For example, a student gives a number, say 8, and the teacher responds with 12. Another student says 6, and the teacher responds 10. As the students begin to recognize the rule, they should raise their hands but not call out the rule. Roles are reversed now, so the teacher says 9 and asks a specific student for the response. If the student says 13, the teacher may simply say, "John knows my rule." An answer other than 13 may result in the teacher saying, "You must be thinking of a different rule," or "You don't know my rule yet." Continue the process until most students know the rule. Then ask one student to identify the rule: "Add 4 to the number."

The game becomes interesting as the students begin to find different rules to fit the pattern.

Example:

Given Number (N)	Response
2	3
5	9
13	25
3	5

Answer: "Double the number, then subtract 1," OR "Add the number to the first whole number smaller than it."

Depending upon the ability of the class, the teacher

may simply accept both rules in verbal form, or write the rules in general terms, or analyze the rules to show why they are equivalent. In the example above, if the *variable* N represents the given number, then the rules are expressed in general terms as "2N-1" and as "N+(N-1)."

Example:

Given Number (N)	Response
3	12
4	20
1	2
9	90

Answer: "A number times its successor," OR, "Add the number and the square of the number." In general terms, "N times (N+1)," or "N^2+N."

"Guess My Rule" is best presented in small doses, both to vary the activities in a class lesson and to expand the idea to other number concepts in this curriculum.

This activity easily leads to the introduction of *sequences*. In a sequence, the first, second, third, and so on, terms merely represent the responses to the given numbers 1, 2, 3, and so on.

Example:

Given Number	Response
1	3
2	6
3	9

The following sequence is formed: 3, 6, 9, . . .

Example: Copy the sequence and fill in the missing terms indicated by blanks.

3, 6, 9, ____, ____, ____, 21, ____, . . .

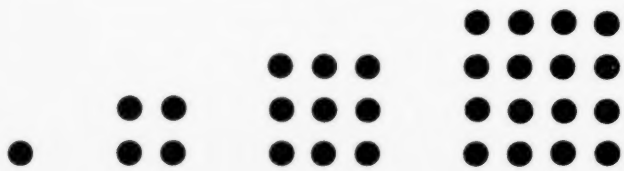
Answer: 3, 6, 9, *12*, *15*, *18*, 21, *24*, . . .

Optional: Any discussion of a general rule for a sequence.

The problem in defining a rule develops from two possible meanings for the general term N. For example, one student might look at the sequence just written, see the 3 times table, and write the rule "3 times N," or 3N. Another student might see that 3 is added to the previous term and state the rule N + 3. The teacher should accept both answers and perhaps mention that N has a different meaning in each case. However, avoid any detailed discussion of writing general rules at this level.

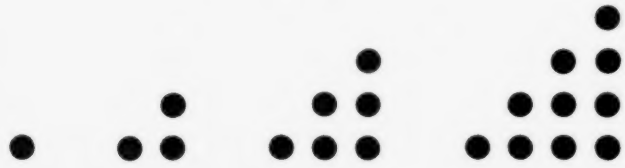
The students should be introduced to these four basic sequences: square numbers, triangular numbers, cubic numbers, and Fibonacci numbers. These sequences offer both geometric representations and patterns that will be important to the students' future study of mathematics. They provide an opportunity to work with whole numbers from a different perspective.

Square numbers: 1, 4, 9, 16, 25, 36, . . . N^2

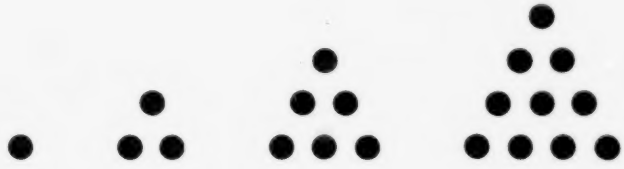


and so on

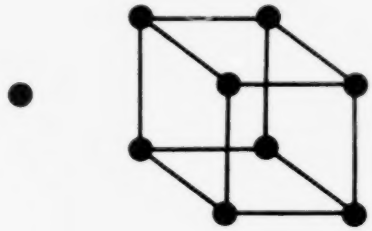
Triangular numbers:
1, 3, 6, 10, 15, 21, 28, . . . $\frac{N(N+1)}{2}$



OR



Cubic numbers: 1, 8, 27, 64, 125, 216, . . . N^3

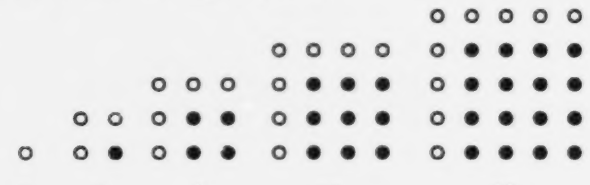


Fibonacci numbers: 1, 1, 2, 3, 5, 8, 13, 21, . . .

(The sum of two consecutive terms equals the next larger term.)

Optional: All work beyond these sequences, including the discovery of patterns found by operating on their terms, should be optional.

Exploration 1: Investigate square numbers through the addition of "L-shaped" pieces.



1 4 9 16 25

or 1^2 2^2 3^2 4^2 5^2

a) What are the next 2 numbers?

b) The dots (•) show how to get from 1 square to the next. How many dots (•) are added each time, starting from 1?

$2^2 = 1^2 +$ ____

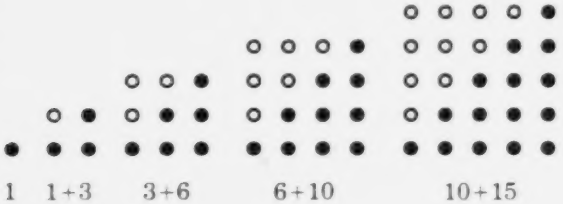
$3^2 = 2^2 +$ ____

$4^2 = 3^2 +$ ____

c) Another pattern is shown by the dots:

1	$= 1 = 1^2$
1 + 3	$= 4 = 2^2$
1 + 3 + 5	= ____ $= 3^2$
1 + 3 + 5 + 7	= 16 = ____
1 + 3 + 5 + 7 + 9	= ____ = ____

Exploration 2: Study the addition of triangular numbers through geometric representations.



1 1+3 3+6 6+10 10+15

Exploration 3: Ask the students to investigate the:

a) Sum of consecutive triangular numbers

b) Sum of squares of consecutive triangular numbers

c) Difference of squares of consecutive triangular numbers

Exploration 4: Have the students write the first 15 Fibonacci numbers.

a) Square any Fibonacci number. Compare the results with the product of its neighbors:

1, 1, 2, 3, 5, 8, 13, . . .

$3 \times 3 = 9$ and $5 \times 2 = 10$

$8 \times 8 = 64$ and $5 \times 13 = 65$

Do the results always differ by one?

b) Add any group of Fibonacci numbers beginning with the first one. Add 1 to the result. What do you notice about the answers?

1 + 1 + 2 + 3 = 7, 7 + 1 = 8, the 6th Fibonacci number

1 + 1 + 2 + 3 + 5 + 8 = 20, 20 + 1 = 21, the 8th Fibonacci number

1 + 1 + 2 + 3 + 5 + 8 + 13 = 33, 33 + 1 = 34, the 9th Fibonacci number

Exploration 5: Let w be a whole number greater than 0 but less than 7. If the cube of w is also a square number, find all values of w.

Answer: 1 and 4

5. Factors and Exponents

Factors of a product are numbers which, when multiplied, give the product. Factors are limited here to the set of natural numbers. Since $2 \times 4 = 8$, then 2 and 4 are factors of 8. The example $1 \times 8 = 8$ illustrates that the number itself and 1 are always factors of the given number. It is also true that $2 \times 2 \times 2 = 8$, or $2 \cdot 2 \cdot 2 = 8$. The raised dot is introduced as a symbol of multiplication to avoid confusion later in the curriculum with the variable x.

When a factor is repeated, as in $2 \cdot 2 \cdot 2 = 8$, the product can be expressed in exponential form, namely $2^3 = 8$. In this example:

a. The *base* (2) is the factor being repeated.

b. The *exponent* (3) indicates the number of times the base is a factor.

c. The *power* (8) is the product of the repeated factors.

The expression 2^3 is read as "2 cubed," "2 raised to the third power," or "the third power of 2," where the third power of 2 is equal to 8.

A number raised to the *first power*, as in $10^1 = 10$ and $3^1 = 3$, is equal to the number itself. The use of patterns, rather than a proof, can help to explain the *zero power* of a natural number.

Example:

$$5 \cdot 5 \cdot 5 = 5^3 = 125$$
$$5 \cdot 5 = 5^2 = 25$$
$$5 = 5^1 = 5$$
$$5^0 = 1$$

$125 \div 5 = 25$, or $5^3 \div 5^1 = 5^2$
$25 \div 5 = 5$, or $5^2 \div 5^1 = 5^1$
$5 \div 5 = 1$, or $5^1 \div 5^1 = 5^0$

In general, for any natural number N, we define $N^0 = 1$.

The various forms used to write a number in the K-6 *Syllabus* should be reviewed and expanded to include the zero exponent.

a. *Standard notation* 3075

b. *Polynomial or expanded form*:
$(3 \times 1000) + (0 \times 100) + (7 \times 10) + (5 \times 1)$

c. *Exponential form*:
$(3 \times 10^3) + (0 \times 10^2) + (7 \times 10^1) + (5 \times 10^0)$

Practice should be provided in working with powers of whole numbers, to include *positive integral exponents* and *zero exponents* (except 0^0). Just as the order of the numbers is important to some operations, such as subtraction, so too is order important in the operation of raising to a power:

$1^8 = 1$, and $8^1 = 8$ $\qquad$ $12^0 = 1$, and $0^{12} = 0$

Example: List the set of all factors of 12.

Answer: {1, 2, 3, 4, 6, 12}

Example: Express $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$ as:
a) a power of 3; b) a power of 9; c) a power of 27.

Answer: a) 3^5 b) 9^3 c) 27^2

Example: I am a factor of 24, but not a factor of 36. I contain only one digit. Who am I?

Answer: 8

Example: Find all possible whole numbers less than 50 that have exactly 3 factors each.

Answer: 4, 9, 25, and 49

6. Prime and Composite Numbers

A *prime number* is a whole number greater than one, whose only factors are 1 and the number itself. A *composite number* is a whole number greater than one, which is not prime. In other words, a composite number has at least one factor that is neither 1 nor the number itself.

In the K-6 *Syllabus*, for grade 4 it is suggested that prime numbers less than 100 be explored on charts such as the Sieve of Eratosthenes, and that prime factors be found for numbers less than 50. No formal study of prime numbers, however, appears in the *Syllabus* for grades 5 and 6, other than its use in finding greatest common factors.

The students should *find* the prime numbers less than 100:

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

The students should also be able to determine whether given numbers are prime or composite. Divisibility rules, mentioned in a later subsection of this unit, may be introduced at this time.

The *prime factors* of a number are all the factors of the given number that are also primes. For example, 3 and 5 are the prime factors of 45. The prime factorization of a number, however, occurs when the number is expressed as a product of prime numbers only. Thus, the prime factorization of 45 is stated as $3 \times 3 \times 5$, or $3 \cdot 3 \cdot 5$.

Practice should be provided in distinguishing between the various types of questions involving factors and primes. For example, list:

a) pairs of factors of 40: 1×40 $\quad$ 4×10
2×20 $\quad$ 5×8

b) the factors of 40: 1, 2, 4, 5, 8, 10, 20, 40

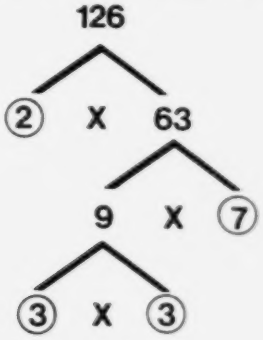
c) the prime factors of 40: 2, 5

d) the prime factorization of 40: $2 \times 2 \times 2 \times 5$, or $2 \cdot 2 \cdot 2 \cdot 5$, or $2^3 \cdot 5$

There are various methods of finding the prime factorization of a number; the choice of presenting one or more methods is left to the teacher.

Example: Express 126 in prime factorization form.

Method 1: Use factor trees. Continue to find pairs of factors for all composite numbers until only prime factors are present. As an aid, circle each prime when found.



Answer: $2 \times 3 \times 3 \times 7$

Method 2: Use repeated division by prime numbers. If possible, divide by the smallest prime number, 2. If possible, divide the quotient by 2 again. If this is not possible, try to divide the quotient by 3. Continue this process, dividing repeatedly by the same prime or by larger primes, until the quotient is 1. The divisors, when multiplied, show the prime factorization.

$$
\begin{array}{r}
1 \\
7\overline{)7} \\
3\overline{)21} \\
3\overline{)63} \\
2\overline{)126}
\end{array}
$$

Answer: $2 \times 3 \times 3 \times 7$

7. Multiples and Divisors

The *multiples* of a natural number n are those numbers which can be shown to have n as a factor. At this point, limit sets of multiples to the natural numbers. For example, the natural number multiples of 6 are 6, 12, 18, 24, 30, . . .

The *divisors* of a natural number n are those natural numbers which are factors of n. For example, the divisors of 6 are 1, 2, 3, and 6.

In grades 4, 5, and 6, the students learned how to find the greatest common factor (GCF) and the least common multiple (LCM) of sets of natural numbers. In grade 7, additional practice should be provided with these topics; the choice of a single method or multiple approaches is left to the teacher.

a. *Greatest common factor*

The greatest common factor (GCF) of two or more numbers is the largest factor that is common to the given numbers.

Example: Find the greatest common factor of 20 and 28, or find GCF (20, 28).

Method 1: List the factors of each number, and select the largest common factor.

Factors of 20: 1, 2, 4, 5, 10, 20

Factors of 28: 1, 2, 4, 7, 14, 28

Notice that 1, 2, and 4 are all common factors, but the largest or greatest of these is 4.

Answer: 4

Method 2: Find the prime factorization of each number. Then the product of all common prime factors is the GCF.

$$20 = 2 \cdot 2 \cdot 5$$
$$28 = 2 \cdot 2 \cdot 7$$

Answer: GCF (20, 28) = $2 \cdot 2 = 4$

Optional: Two numbers are said to be relatively prime to each other if their greatest common factor is 1. For example, 6 and 25 are relatively prime.

b. *Least common multiple*

The least common multiple (LCM) of two or more numbers is the smallest natural number that is a common multiple of the given numbers.

Example: Find the least common multiple of 6 and 8, or find LCM (6, 8).

Method 1: List the multiples of each number, limited to the set of natural numbers, and select the smallest common multiple.

Multiples of 6: 6, 12, 18, 24, 30, 36, 42, 48, . . .

Multiples of 8: 8, 16, 24, 32, 40, 48, . . .

Notice that 24, 48, 72, 96, . . . are common multiples, but the smallest or least of these is 24.

Answer: 24

Method 2: Find the prime factorization of each number. The product of the GCF and all other listed factors of the given numbers is the LCM. (The process of finding the LCM is aided by using columns to indicate duplicate or common factors, such as the GCF of 2 as shown below.)

$6 = 2 \cdot 3$

$8 = 2 \cdot 2 \cdot 2$

Answer: $LCM(6, 8) = 2 \cdot 3 \cdot 2 \cdot 2 = 24$

The students are familiar with the set operations of intersection and union as described in the K-6 *Syllabus*. Such operations can be integrated with GCF and LCM as an alternate approach.

Recall that an *intersection* of two sets, A and B, is the set of all elements common to both set A and set B, indicated in symbols as $A \cap B$. Note that $A \cap B$ is read as "A and B" or as "A intersection B." A *union* of two sets, A and B, is a set obtained by grouping together the elements of set A with the elements of set B without repeating elements common to both sets. The symbols, $A \cup B$, are read as "A or B" or as "A union B."

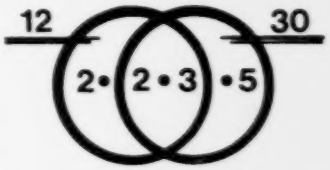
Example: $A = \{10, 20, 30\}$

$B = \{20, 40\}$

Intersection: $A \cap B = \{20\}$

Union: $A \cup B = \{10, 20, 30, 40\}$

Just as the intersection of two sets deals with those elements that are common to both sets, so does the GCF deal with the product of the listed factors common to the prime factorization of two or more numbers. In a similar way, the union of sets is related to the LCM of two or more numbers. Study the following set diagram to see the relationships.



$12 = 2 \cdot 2 \cdot 3$

$30 = 2 \cdot 3 \cdot 5$

Intersection ⟷ $GCF(12, 30) = 2 \cdot 3 = 6$

Union ⟷ $LCM(12, 30) = 2 \cdot 2 \cdot 3 \cdot 5 = 60$

Practice should be provided in finding the GCF and LCM of more than two numbers.

Example: $GCF(28, 42, 70) = 14$

$LCM(28, 42, 70) = 420$

c. Rules for divisibility

As an aid in factoring, the students should know the rules for divisibility of natural numbers by 2, 3, 5, and 10. Indicate that one whole number is divisible by a second whole number if the first number can be divided by the second without a remainder.

- A number is divisible by 2 if the last (or units) digit of the numeral is 0, 2, 4, 6, or 8. Such a number is called an *even* number.
- A number is divisible by 3 if the sum of its digits is divisible by 3. (For example, 87 is divisible by 3 because $8 + 7 = 15$, and 15 is divisible by 3. Similarly, $1 + 5 = 6$, and 6 is divisible by 3.)
- A number is divisible by 5 if the last digit of the numeral is 0 or 5.
- A number is divisible by 10 if the last digit of the numeral is 0.

Optional: Other rules for divisibility are optional, but their investigation may lead to the discovery of many interesting patterns.

- A number is divisible by 4 if the number formed by its tens and units digits is divisible by 4. (For example, 752 is divisible by 4 because 52 is divisible by 4.)
- A number is divisible by 6 if it is divisible by both 2 and 3.
- A number is divisible by 7 if the difference between the number formed by its nonunits digits and twice its units digit is divisible by 7. (For example, 476 is divisible by 7 because $47 - 2 \times 6 = 47 - 12 = 35$, and 35 is divisible by 7.)
- A number is divisible by 8 if the number formed by its hundreds, tens, and units digits is divisible by 8. (For example, 5240 is divisible by 8 because 240 is divisible by 8.)
- A number is divisible by 9 if the sum of its digits is divisible by 9. (For example, 5337 is divisible by 9 because $5 + 3 + 3 + 7 = 18$, and 18 is divisible by 9.)

Problem-solving skills can be applied to variations of the standard examples involving multiples and divisors, to reenforce techniques learned earlier and to consider generalizations on a simple level.

Example: Choose any whole number. Rearrange the digits in any way. Subtract the smaller from the larger:

a. 3541 1543

$$\begin{array}{r} 3541 \\ -1543 \\ \hline 1998 \end{array}$$ or 222×9

b. 813 138

$$\begin{array}{r} 813 \\ -138 \\ \hline 675 \end{array}$$ or 75×9

Is the answer always divisible by 9? Try many numbers.

Answer: Yes

Example: Is the GCF of two even numbers also an even number? Why?

Answer: Yes. There is a common factor of 2, and any multiple of 2 is an even number.

Example: Find the LCM of the divisors of 200.

Answer: 200

(In general, the LCM of the divisors of x is x.)

Example: If LCM (12, x) = 36, find all possible values of x.

Answer: 9, 18, 36

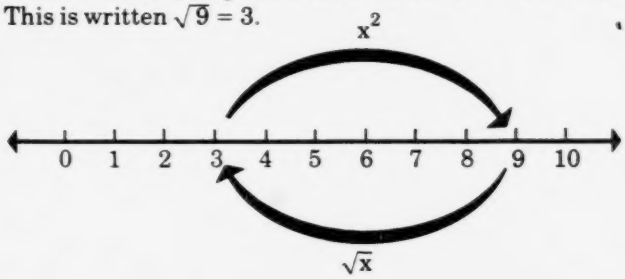
Example: Find the five smallest whole number values of x for which the statement GCF (8, x) = 2 is true.

Answer: 2, 6, 10, 14, 18

(This can be expanded to questions about sequences.)

8. Principal Square Roots that are Whole Numbers

The concept of "square root" can be considered as an *inverse* operation of "squaring" when we restrict the set being considered to whole numbers only. To indicate a square root of a number, we use a *radical sign*, $\sqrt{\ }$. For example, the square of 3 is 9 because $3^2 = 3 \cdot 3 = 9$. In a similar manner, a square root of 9 is 3 because $3 \cdot 3 = 9$. This is written $\sqrt{9} = 3$.

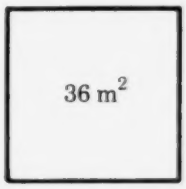


At this level, numbers should be restricted to perfect-square radicands. A *radicand* is a number under the radical sign. For example, in $\sqrt{16}$, read as "a square root of 16," or as "radical 16," the radicand is 16.

Simple activities can include the construction of a chart of squares of whole numbers from 0 through 20. Square roots of these squares can then be studied.

Number (n)	Square (n•n, or n^2)
0	0
1	1
2	4
3	9
. . .	. . .
20	400

Example: A square garden has an area of 36 square meters. Find the length of a side.
(Hint: Which two equal factors give 36?)



Answer: 6 meters

Example: Find the value of: a) $\sqrt{81}$ b) $\sqrt{256}$

Answer: a) 9 b) 16

The rules of divisibility learned earlier, along with some notion of square root, provide clues in determining whether a number is prime or composite in relatively few steps.

For example, 100 is a composite number. Pairs of factors of 100 are listed here. As the factors at the left increase, the factors at the right decrease until 10×10 is reached. Remember that $\sqrt{100} = 10$.

1×100
2×50
4×25
5×20
10×10

Example: Is 167 a prime or a composite number?

Since 167 is close to 169, and $\sqrt{169} = 13$, try to divide 167 by the prime numbers only from 2 through 13. Since 167 is not divisible by 2, 3, 5, 7, 11, or 13, we can claim that 167 is prime. Why? If these smaller numbers are not divisors, then there are no larger numbers that are divisors. Also, since $13 \times 13 = 169$, we appear to have passed the "turn-around" point for pairs of factors of 167.

Answer: 167 is prime.

9. Order of Operations

In the K-6 *Syllabus*, the order of operations with whole numbers was introduced in grade 5 but does not appear in grade 6. The students will require practice with this topic, but care should be taken to construct examples so that fractions and negative values are avoided. Problem-solving situations wherein the order of operations is important should be presented.

The order of operations used to simplify a numerical expression is given as follows:

(1) Work within parentheses first. If more than one

symbol of grouping is given, simplify the innermost grouping first.

(2) Next, simplify all powers (or exponents).

(3) Then, perform all multiplications and divisions, in order from left to right.

(4) Lastly, perform all additions and subtractions, in order from left to right.

Example: Simplify each expression:

a.	$3+5\times2$	b.	$10-3^2$	c.	$(10-3)^2$
	$3+10$		$10-9$		$(7)^2$

Answer:

a. 13 b. 1 c. 49

Example: In a game, Len had 15 points, Fred had 13, and Lee had 6. Two more rounds were played. Len lost 5 points, then doubled his score. Fred doubled his score, then lost 5 points. Lee gained 3 points, then doubled that score. Who won the game?

Len: $(15 - 5) \times 2 = 10 \times 2 = 20$
Fred: $13 \times 2 - 5 = 26 - 5 = 21$
Lee: $(6 + 3) \times 2 = 9 \times 2 = 18$

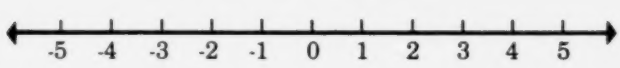
Answer: Fred

B. INTEGERS

Negative numbers were studied in grade 6; the students were introduced to addition of integers on the number line. In grade 7, the students will learn for the first time how to subtract, multiply, and divide integers. In grade 8, these operations will be studied again, along with a formal introduction to their properties.

1. Definition

Every natural number (also called a positive integer) has an *opposite* which is called a negative integer. The opposite of 3 is -3. The positive and negative integers, combined with zero, form the *integers*, 0, 1, –1, 2, –2, 3, –3,. . . . The set of integers can be displayed on a number line and listed in a corresponding manner.



The set of integers = {. . ., –3, –2, –1, 0, 1, 2, 3, . . .}

The natural number 4 and the positive integer +4 name the same number. That is, +4 = 4. The opposite of +4 and the opposite of 4 also name the same number, or –(+4) = –4. The symbol, –4, has two meanings: negative 4, or the opposite of 4.

The students should experience situations involving opposites, and relate these applications to the number line: right/left; win/lose; east/west; north/south; up/down; increase/decrease; forward/backward; gain/loss; and so forth.

Example: What is the opposite of going 5 km south?

Answer: Going 5 km north

Example: Locate an integer 7 units left of zero on the number line. What is its opposite?

Answer: 7, or +7

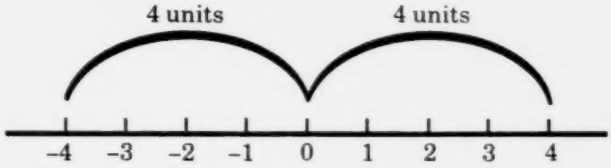
Example: I am an integer and I am my own opposite. Who am I?

Answer: 0

2. Absolute Value

For every nonzero integer, there is always a second integer called its opposite. The opposite of 8 is –8, and the opposite of –8 is 8. The *absolute value* of a number is written in symbols by placing a pair of vertical bars, | |, about the number. From an *arithmetic* viewpoint, the greater value of a nonzero number and its opposite is called the absolute value of the number. Thus, $|8| = 8$, and $|-8| = 8$, because 8 is the greater value of 8 and –8.

From a *geometric* viewpoint, the absolute value of a number is the distance between two points on a number line, one representing zero and another representing the number. In this way, $|-4| = 4$ because the point for –4 is 4 units from the point representing 0. Also, $|4| = 4$.



It should be stated that the absolute value of zero is zero, written $|0| = 0$. The students should realize that the absolute value of any given integer is either positive or zero.

Symbols of equality and *inequality* are used to compare quantities. The following symbols should be discussed and applied to a comparison of integers and absolute values:

"is equal to" =
"is greater than" >
"is less than" <

The students should understand that on a number line, in standard horizontal form, the positive numbers are to the right of 0, the negative numbers are to the left of 0, and the greater of any two numbers is always found at the point further to the right.

Example: Between each pair of numbers, insert one of the symbols (=, >, <) to make the statement true.

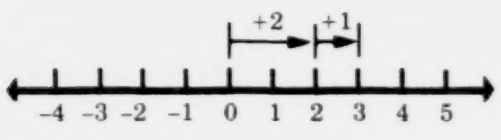
a) 5 −8	b) −5 −8	c) 5 |−8|
d) |−3| 3	e) |−6| −6	f) |−2| |2|

Answer:

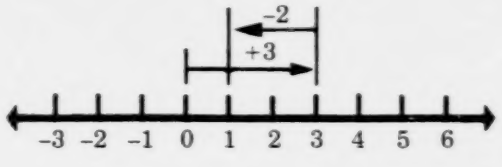
a) 5 > −8	b) −5 > −8	c) 5 < |−8|
d) |−3| = 3	e) |−6| > −6	f) |−2| = |2|

3. Addition of Integers

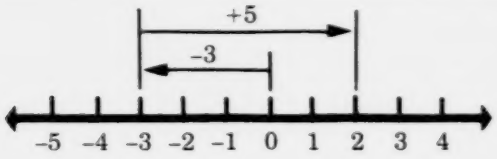
Addition of integers can be related to a series of moves on the *number line*. Moving to the right is related to a positive number, and moving to the left is related to a negative number. In all cases, the starting point is 0.



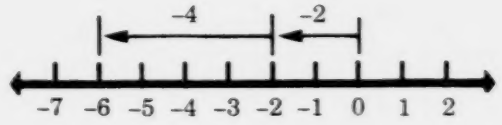
(+2) + (+1) = +3



(+3) + (−2) = +1



(−3) + (+5) = +2



(−2) + (−4) = −6

The examples just written in horizontal form should also be shown in vertical form.

Add:	+2	+3	−3	−2
	+1	−2	+5	−4
	+3	+1	+2	−6

The students should realize that counting on a number line becomes impractical with larger values, or values further from zero. Encourage the class to develop the necessary rules to add any two integers:

- To add two numbers having the same sign, add the absolute values of the numbers and write the common sign before this sum. (Two rules, one for addition of positive numbers and one for addition of negative numbers, may also be stated.)
- To add two numbers having different signs, subtract the absolute values of the numbers and write the sign of the number whose absolute value is greater before this difference. (Again, variations of this rule are possible.)

Check the rules formulated by the students with some examples.

Add:		*Think*:	*Write*:
	−97	97 − 85 = 12	−97
	+85	Since |−97| > |+85|, the	+85
		answer is negative.	

Answer: −12

a. Subtraction of integers

Subtraction of integers may be presented in any one or more of several ways, including:

1) A definition of subtraction, namely, $a - b = c$ if and only if $c + b = a$. Therefore, 10 − 2 = 8 because 8 + 2 = 10.

To solve $2 - 5 = \square$, think $\square + 5 = 2$. Thus, $2 - 5 = -3$.

To solve $5 - (-3) = \square$, think $\square + (-3) = 5$. Here, $5 - (-3) = 8$.

2) The use of patterns, such as:

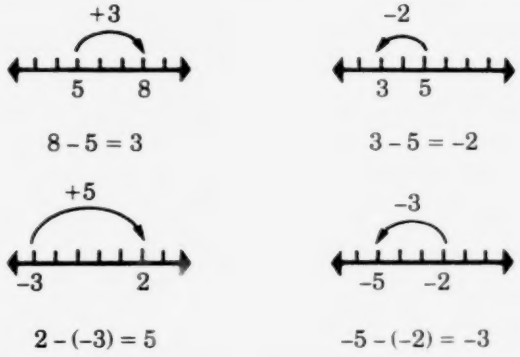
$6 - 3 = 3$	$6 - 3 = 3$
$6 - 2 = 4$	$6 - 4 = 2$
$6 - 1 = 5$	$6 - 5 = 1$
$6 - 0 = 6$	$6 - 6 = 0$
$6 - (-1) =$	$6 - 7 =$
$6 - (-2) =$	$6 - 8 =$

3) The concept of opposites. For example, $-(+3)$ or $-(3)$ may be thought of as "subtracting 3" or as "the opposite of 3." In either case, $-(+3) = -3$, and $-(3) = -3$. Similarly, $-(-4)$ may be regarded as "subtracting negative 4" or as "the opposite of negative 4." In this case, $-(-4) = 4$ or $-(-4) = +4$. Thus, subtracting -4 is equivalent to adding 4.

$6-(+3)=6-3=3$, or $6-(+3)=6+(-3)=3$

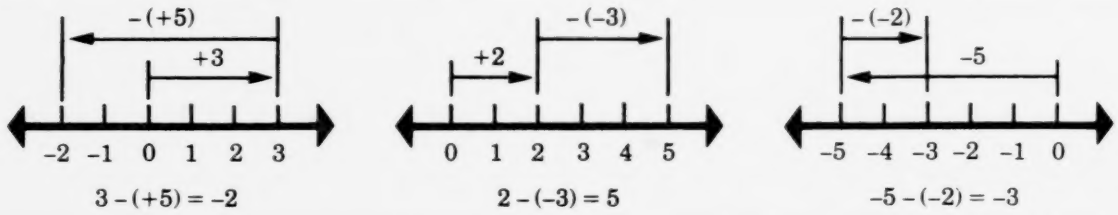
$9-(-4)=9+4=13$, or $9-(-4)=9+(+4)=13$

4) The use of a number line, wherein the difference is thought of as a directed distance from the subtrahend to the minuend. (Minuend – subtrahend = difference)



5) A combination of the number line and the use of opposites. If $+3$ is shown as ⟼, then $-(+3)$ can be ⟻.

Also, if -4 is ⟻, then $-(-4)$ can be ⟼.



Regardless of the approach used, the students should eventually realize that subtraction is simply the inverse operation of addition, and they should be encouraged to formulate the following rule (or some variation of the rule):

To subtract two numbers, change the sign of the subtrahend (the number being subtracted) and follow the rules for addition.

b. Problem solving

The teacher can supplement computational drill with simple problem-solving situations.

Example: In two plays, a football team lost 7 yards and gained 4 yards. What was the net result of the 2 plays?

$(-7) + (+4) = -3$

Answer: A loss of 3 yards

Example: At noon, the temperature was 10°C. At 8:00 p.m., the temperature was 6°C. What was the change in the temperature?

$6 - 10 = -4$

Answer: –4°C, or a drop of 4 degrees Celsius.

Example: In a game, David gained 3 points, then lost 10. Mary lost 2 points, then lost 4 more. Who has the higher score? By how much is that person winning?

David: $(+3)+(-10)=-7$

Mary: $(-2)+(-4)=-6$

$-6 > -7$

$(-6)-(-7)=-6+7=1$

Answer: Mary has the higher score, by 1 point.

4. Multiplication of Integers

Multiplication of integers can be presented using any mathematical procedure acceptable to the teacher. Only a few approaches are shown here.

- The use of patterns, such as:

$(+3)(+3)$	$=9$	$(-3)(+3)$	$=-9$
$(+3)(+2)$	$=6$	$(-3)(+2)$	$=-6$
$(+3)(+1)$	$=3$	$(-3)(+1)$	$=-3$
$(+3)(0)$	$=0$	$(-3)(0)$	$=0$
$(+3)(-1)$	$=$	$(-3)(-1)$	$=$
$(+3)(-2)$	$=$	$(-3)(-2)$	$=$
$(+3)(-3)$	$=$	$(-3)(-3)$	$=$

- The use of story situations. For example, if +3 represents 3 weeks in the future, then –3 represents 3 weeks ago. If +4 indicates that Jennifer saves $4, then –4 indicates that Jennifer spends $4.

  (1) If Jennifer saves $4 each week, then 3 weeks from now she will have saved $12. $(+3)(+4)=+12$

  (2) If Jennifer spends $4 each week, then 3 weeks from now she will have spent $12. $(+3)(-4)=-12$

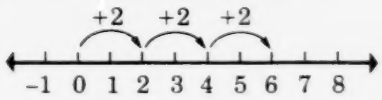
  (3) If Jennifer saves $4 each week, then 3 weeks ago she had $12 less in her bank than she has now. $(-3)(+4)=-12$

  (4) If Jennifer spends $4 each week, then 3 weeks ago she had $12 more than she has now. $(-3)(-4)=+12$

- The use of a number line, where multiplication is shown as repeated addition.

  (1) Think of $(+3)(+2)$ as 3 groups of +2.

  $(+3)(+2) = (+2) + (+2) + (+2)$

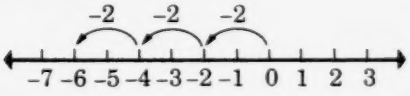


  or

  $(+3)(+2)= +6$

  (2) Think of $(+3)(-2)$ as 3 groups of –2.

  $(+3)(-2)=(-2)+(-2)+(-2)$

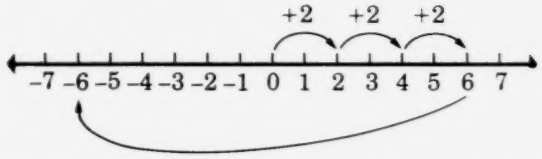


  or

  $(+3)(-2)=-6$

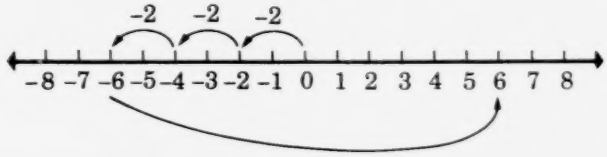
  (3) Since +3 means 3 groups, then –3 means the *opposite* of 3 groups. Think of $(-3)(+2)$ as the opposite of 3 groups of +2. That is, take 3 groups of +2, then find its opposite.

  $(-3)(+2)=-(3)(+2)$
  $=-6$



  (4) Think of $(-3)(-2)$ as the opposite of 3 groups of –2. That is, take 3 groups of –2, then find its opposite.

  $(-3)(-2)=-(3)(-2)$
  $-(-6)$
  $=+6$



The students should again be encouraged to find the rules, this time for the multiplication of any two integers. Variations of the rules stated here are acceptable:

(1) To multiply two numbers having the same signs, multiply their absolute values. (The product is positive.)

(2) To multiply two numbers having different signs, multiply their absolute values and write a negative sign before the product.

(3) The product of any number and zero is 0.

---

a. Division of integers

Division of integers may be treated quickly as an inverse operation of multiplication: $a \div b = c$ if and only if $c \cdot b = a$.

To solve $(+12) \div (+2) = \square$, think $\square \cdot (+2) = +12$. Thus, $(+12) \div (+2) = +6$.

To solve $(+8) \div (-2) = \square$, think $\square \cdot (-2) = +8$. Thus, $(+8) \div (-2) = -4$.

To solve $(-9) \div (+3) = \square$, think $\square \cdot (+3) = -9$. Thus, $(-9) \div (+3) = -3$.

To solve $(-12) \div (-2) = \square$, think $\square \cdot (-2) = -12$. Thus, $(-12) \div (-2) = +6$.

The rules for division of integers parallel the rules for multiplication, except that division by 0 is not possible.

b. Problem solving

Computational drill should again be supplemented by problem-solving situations involving the multiplication and division of integers.

Example: Joanna was on a hiking trip. Every hour, she reached an altitude that was 50 feet higher than the previous hour. If Joanna was at an altitude of 820 feet at 3:00 p.m., at what altitude was she at noon (3 hours earlier)?

$(-3)\bullet(50)=-150$
$820-150=670$

Answer: 670 feet

Example: Larry went into business for the summer vacation, but it rained often. Over 8 weeks, his total expenses were \$320, and he took in only \$120. Find Larry's average weekly profit or loss.

$(-320)+(120)=-200$
$(-200)\div 8=-25$

Answer: –\$25, or an average weekly loss of \$25.

5. Order of Operations

Examples involving the *order of operations* (studied with whole numbers in II–A–9) can be extended to include answers that are negative integers. Exercises should be kept at a relatively simple level, and problem-solving situations should be presented to reflect negative quantities.

Example: Simplify the expression:

$21\div 3-6\times 2$
$7-12$
-5

Answer: –5

Example: Every month, Mrs. Arsena deposits \$35 of her pay check into her account. Four times each year, she withdraws \$60 for a special dinner or a night out. What does she save over a period of one year?

$(12)(+35)+(4)(-60)=+420+(-240)=+180$

Answer: \$180

C. RATIONAL NUMBERS

Operations with Fractions is one of the strands in the K-6 *Syllabus*. In those grades, the students were taught to add, subtract, multiply, and divide both common fractions and decimal fractions. Fractional equivalents, decimal-fraction equivalents, and concepts of order (greater than, less than, between) were also studied.

In grade 7, although problem solving serves as the basis of this unit, many students will require practice in computation with rational numbers. The extent to which the algorithms for computation are reviewed or retaught, however, is dependent upon the needs of the class and the ability levels of individual students.

The use of alternate algorithms may appeal to some students, and may offer others a new approach to a familiar topic. Certain concepts not covered in K-6, such as exponents, sequences, and order of operations, should provide a means for computational drill within a new framework.

In grade 7, although the students will be required to recognize negative as well as positive rational numbers, all computation will be restricted to the set of positive rational numbers. Computation involving negative rational numbers will be delayed until grade 8.

1. Definition and Subsets of the Rational Numbers

Now that the students have studied the set of integers, the teacher can present the complete definition of a rational number. A *rational number* is any number which can be expressed as the quotient of two integers, a/b, where $b \neq 0$. In other words, a/b is equivalent to a divided by b.

The students should be able to identify numbers that are rational, including natural numbers, zero, and negative integers, and write them in the form established by the definition. For example:

$8 = 8/1 = 16/2 = \ldots$ $\quad 0 = 0/1 = 0/2 \ldots = \ldots$ $\quad -5 = -5/1 = -10/2$

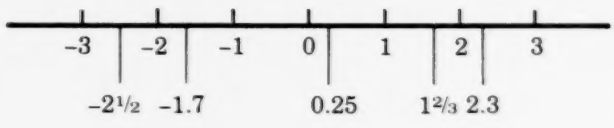
Other numbers, including mixed numbers and decimal fractions, should also be recognized by the students as rational numbers.

$5\frac{1}{2} = 5 + \frac{1}{2} = \frac{10}{2} + \frac{1}{2} = \frac{11}{2}$ $\quad 0.4 = \frac{4}{10} = \frac{2}{5}$

At this time, however, some students might not be able to find the equivalent form stated in the definition for some rational numbers. This can serve as motivation for future work, since procedures to find equivalent forms will be discussed within this unit.

The number line should be presented, and the students should realize that every rational number has a position or corresponding point on this line.

Example:



a) How many of the twelve numbers represented on the number line are rational numbers?

b) Write each of these rational numbers in the form $^{a}/_{b}$ where a and b are integers, and b ≠ 0.

Answer: a) All twelve are rational.

b) Answers will vary.

$-3 = {}^{-3}/_{1}$ $-2\frac{1}{2} = {}^{-5}/_{2}$ $-2 = {}^{-2}/_{1}$ $-1.7 = {}^{-17}/_{10}$

$-1 = {}^{-1}/_{1}$ $0 = {}^{0}/_{3}$ $0.25 = {}^{1}/_{4}$ $1 = {}^{6}/_{6}$

$1\frac{2}{3} = {}^{5}/_{3}$ $2 = {}^{10}/_{5}$ $2.3 = {}^{23}/_{10}$ $3 = {}^{3}/_{1}$

Example: Using any two integers from {0, 1, 2, 4} to replace a and b in the form $^{a}/_{b}$, how many different rational numbers can be formed? (Hint: $^{0}/_{1}$ and $^{0}/_{2}$ both name the same rational number, 0.) Place the numbers on the rational number line.

Answer: There are 5 different rational numbers.

A brief discussion of subsets is appropriate at this time. The largest set under consideration is called the *universal set*, or U. When elements of the universal set are grouped in various ways to form several sets, these sets are called *subsets* of U. Let the universal set be the set of rational numbers. Subsets of the rational numbers include the set of whole numbers, the set of integers, the set of positive rationals, and so forth. The students might need to be reminded that every set is a subset of itself, and that the null set or empty set is a subset of any universal set. All numbers that the students have experienced to date in the curriculum can be defined as various subsets of the rational numbers.

The complement of a set, which is a concept often used in critical thinking, should also be discussed. For any given subset A of a universal set U, the *complement* of set A (sometimes called "not A") is the set of elements of the universal set not found in A.

For example, if U = {rational numbers}, then the complement of the negative rational numbers is the set of positive rational numbers *and* zero. The union of any set and its complement is always the universal set, and the intersection of any set and its complement is always the empty set.

Any set may be identified as the universal set. Let U = {positive fractions}. The set of *proper fractions* is the set of all fractions greater than zero but less than one, such as $^{1}/_{8}$ and $^{299}/_{300}$. The complement of the set of proper fractions is the set of *improper fractions*, that is, all fractions equal to or greater than one, such as $^{3}/_{3}$, $^{5}/_{2}$, and $^{200}/_{7}$.

Similarly, let U = {integers}. The set of *even numbers* is the set of all numbers divisible by 2, namely {. . ., –4, –2, 0, 2, 4, ...} The complement of the even numbers is the set of *odd numbers*, that is, the set of all integers not divisible by 2, or {. . ., –3, –1, 1, 3, ...}

Example: I am a positive rational number but not an improper fraction. My numerator and denominator are prime numbers less than 5. Who am I?

Answer: $^{2}/_{3}$

Example: What integer is not negative, not odd, and not a counting number?

Answer: 0

(What information is extraneous in the above example? "Not odd.")

2. Equivalent Fractions

The students have worked with equivalent common fractions in grades 3-6, including the use of concrete models, the number line, and the identity element 1. Some mention of this topic, however, is important to the operations which follow.

The *terms* of a common fraction such as $^{4}/_{8}$ are the *numerator* 4 and the *denominator* 8. Every rational number may be written in an infinite number of ways, either by reducing to lower terms or by increasing to higher terms. For example: $^{1}/_{2} = {}^{2}/_{4} = {}^{3}/_{6} = {}^{4}/_{8} = {}^{5}/_{10} = {}^{6}/_{12}$ . . . *Equivalent fractions* are any two or more fractions which name the same number.

a. Reducing to lowest terms

To simplify, or to reduce a fraction to lowest terms, find the largest whole number which will divide both terms without a remainder. This number is called the greatest common divisor or the greatest common factor. Both terms of the fraction are then divided by this greatest common factor.

$^{4}/_{8} = {}^{1}/_{2}$ or $\frac{4}{8} = \frac{4\times1}{4\times2} = \frac{4}{4}\times\frac{1}{2} = 1 \times {}^{1}/_{2} = {}^{1}/_{2}$

or $^{4}/_{8} = \frac{4\div4}{8\div4} = {}^{1}/_{2}$

Notice that $^{4}/_{4} = {}^{1}/_{1} = 1$. In the set of rational numbers, 1 is called the *identity element under multiplication* because, when any given number is multiplied by 1, its

value is not changed. That is, $n \cdot 1 = n$, and $1 \cdot n = n$. For this reason, we know that $4/8 = 1/2$, or that $4/8$ and $1/2$ are equivalent fractions.

If the students cannot find the greatest common factor to reduce to lowest terms, they may use prime factorizations or a process of continually reducing to ever lower terms until the fraction is in its simplest form.

Example: $24/180 = \dfrac{2 \cdot 2 \cdot 2 \cdot 3}{2 \cdot 2 \cdot 3 \cdot 3 \cdot 5} = 2/15$

Example: $42/14 = 21/7 = 3/1 = 3$

b. Increasing to higher terms

The identity element 1 is also used to increase a fraction to higher terms, a procedure used to add fractions with unlike denominators.

For example, to find a fraction equivalent to $2/5$ whose denominator is 35, multiply $2/5$ by a form of the identity element, 1, so that the denominator of the product is 35. Here, since $5 \times 7 = 35$, the form $7/7$ is used for the identity element 1. Thus, $2/5$ and $14/35$ are equivalent fractions.

Example: Which two of the following fractions are equivalent?

$14/21 \quad 18/24 \quad 18/27 \quad 15/25 \quad 15/27$

Answer: $14/21 = 18/27$ (Each reduces to $2/3$)

3. Operations and Problem Solving with Positive Common Fractions

a. Four basic operations

The four basic operations with positive common fractions, and zero, should be reviewed through problem-solving situations. The students should see both horizontal and vertical forms for addition and subtraction; the horizontal form should be emphasized for multiplication and division.

To add or subtract fractions with unlike denominators, first find equivalent fractions whose denominators are alike. The identity element 1 is used in the following example, but the teacher may use other approaches.

$$\frac{1}{4} + \frac{2}{3} = \frac{1}{4}\left(\frac{3}{3}\right) + \frac{2}{3}\left(\frac{4}{4}\right) = \frac{3}{12} + \frac{8}{12} = \frac{11}{12}$$

The teacher may present one or several algorithms for the same operation. Here are alternate algorithms used to subtract two mixed numbers:

Regrouping, with the middle step eventually omitted.

$$\begin{aligned} 4\,1/7 &= 3 + 1\,1/7 = 3\,8/7 \\ -2\,5/7 &= -2\,5/7 \quad = -2\,5/7 \\ \hline & \qquad\quad 1\,3/7 \end{aligned}$$

The use of improper fractions.

$$4\,1/7 - 2\,5/7 = 29/7 - 19/7 = 10/7 = 1\,3/7$$

Shifting on the number line to keep the same difference or distance between two points. Thus, the same fraction is added to both minuend and subtrahend so that the subtrahend is made to be a whole number.

$$\begin{aligned} 4\,1/7 \quad & (4\,1/7 + 2/7) = 4\,3/7 \\ -2\,5/7 \quad & -(2\,5/7 + 2/7) = -3 \\ \hline & \qquad\quad 1\,3/7 \end{aligned}$$

The use of signed numbers (recommended only for the more capable students).

$$\begin{aligned} 4\,1/7 &= 4 + 1/7 \\ -2\,5/7 &= -2 - 5/7 \\ \hline & 2 - 4/7 = 1\,7/7 - 4/7 = 1\,3/7 \end{aligned}$$

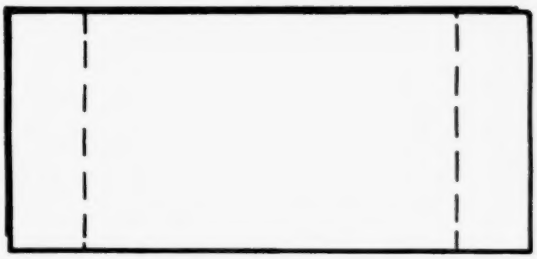
Estimation skills should also be tested with these operations. For example, $2/5 + 3/7$ must name a sum "less than 1" because each addend is less than $1/2$. Similarly, $12 - 7\,2/5$ is "less than 5." The range of estimates depends upon the numbers in the problem.

Example: Nancy spent $1/2$ hour in mowing the lawn, $3/4$ hour in raking, and $1/3$ hour in sweeping. How many hours did Nancy work at these chores?

Answer: $1\,7/12$ hours

(An answer of 1 hour 35 minutes may be checked in several ways.)

Example: Mrs. Macrina bought 6 yards of cloth to make curtains for 3 windows. If each of the two smaller windows requires $5/6$ yard of cloth, how much material is left for the third window?



Answer: $4\,1/3$ yards

Example: To make butter cookies, mix the ingredients listed here, preheat an oven to 300°F, and bake the cookies for 20 to 25 minutes. How many cups of ingredients are needed for this recipe?

1 cup all purpose flour
$1/2$ cup cornstarch
$2/3$ cup powdered sugar
$3/4$ cup butter
$1/2$ cup chopped walnuts

Answer: $3\,5/12$ cups

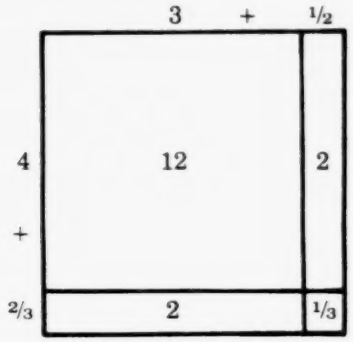
To multiply fractions or mixed numbers, each number is written in the form a/b. Alternate methods may be shown.

$$15/16 \bullet 2/5 = 30/80 = 3/8 \text{ or } \frac{3\bullet 5}{2\bullet 8} \bullet 2/5 = 3/8$$

$$3\,1/2 \times 4\,2/3 = 7/2 \times 14/3 = \frac{7}{2\bullet 1} \times \frac{7\bullet 2}{3} = 49/3 = 16\,1/3$$

To *estimate* the product $3\,1/2 \times 4\,2/3$, the students should realize that the product is greater than 3×4, or 12, and that it is less than 4×5, or 20. Although this range of values is wide, it is helpful as a quick check of the final product.

A geometric model of this same problem shows the product as the sum of four smaller products, each of which represents an area:



Optional: *Exponents* and *principal square roots* (restricted to rational numbers which are perfect squares) provide practice in computation within a new setting.

$$(3/5)^3 = 3/5 \times 3/5 \times 3/5 = 27/125$$

$$\sqrt{81/400} = 9/20$$

$$(2\,3/4)^2 = (11/4)^2 = 121/16 = 7\,9/16$$

$$\sqrt{12\,1/4} = \sqrt{49/4} = 7/2 = 3\,1/2$$

To divide two fractions or mixed numbers, each number is written in the form a/b, and the dividend is then multiplied by the reciprocal of the divisor. Simple examples like $6 \div 2 = 3$ and $6 \times 1/2 = 3$ illustrate the concept that division by a given number is equivalent to multiplication by the reciprocal of that number. The *reciprocal* or *multiplicative inverse* of the rational number a/b is b/a, where $a \neq 0$ and $b \neq 0$. In other words, every number except 0 has a multiplicative inverse. State that the product of any nonzero rational number and its multiplicative inverse is 1, the identity element for multiplication.

$$2/3 \div 3/4 = 2/3 \times 4/3 = 8/9$$

$$6\,7/8 \div 2\,1/2 = 55/8 \div 5/2 = 55/8 \times 2/5 = 11/4 = 2\,3/4$$

From $3 \div 4 = 3 \times 1/4 = 3/4$, it can be seen that the fraction $3/4$ is equivalent to "3 divided by 4."

b. Simplifying complex fractions

This concept can be extended to the simplification of complex fractions. For example:

$$\frac{2\,1/3}{3\,1/6} = 2\,1/3 \div 3\,1/6 = 7/3 \div 19/6 = 7/3 \times 6/19 = 14/19$$

The complex fraction $2\,1/3 \div 3\,1/6$ is written in its simplest form as $14/19$.

The teacher should feel free to show one or more methods to simplify a complex fraction. Here are two alternate algorithms, each using multiplication by some form of the identity element 1.

Method 1: Each term in the identity n/n is the LCM of the denominators of the terms in the complex fraction.

$$\frac{2\,1/3}{3\,1/6} = \frac{7/3}{19/6} \times \frac{6}{6} = 14/19$$

Method 2: Each term in the identity n/n is the reciprocal of the denominator in the complex fraction.

$$\frac{2\,1/3}{3\,1/6} = \frac{7/3}{19/6} = \frac{7/3}{19/6} \times \frac{6/19}{6/19} = \frac{14/19}{1} = 14/19$$

Estimation skills should be tested with division of fractions. The quotient obtained from $2/7 \div 3/5$ should be "less than 1" because the dividend $2/7$ is less than the divisor $3/5$. Similarly, $8\frac{1}{5} \div 3\frac{1}{5}$ yields a quotient "larger than 2 but less than 3" by an inspection of its whole number components.

Example: Mr. Jones earns $5 per hour as his regular pay. He is paid time-and-a-half (or $1\frac{1}{2}$ times his regular pay) for all work over 40 hours each week. If Mr. Jones works 56 hours in one week, what does he earn?

Answer: $320.

Example: The questions on a test all have the same point value. Joan answered $9/10$ of the questions asked, and got $5/6$ of these correct. What part of the total questions on the test did Joan answer correctly?

Answer: $3/4$

(Did Joan pass? How many questions could have been on the test? 20,40,60,. . .)

Example: If a car travels 60 km in $1\frac{1}{4}$ hours, what is its average speed?

$$\text{Rate} = \frac{\text{Distance}}{\text{Time}}$$

Answer: 48 km/hr.

The study of sequences can also be applied to multiplication and division.

Example: A sequence is given as $1/2, 2/3, 3/4, 4/5, 5/6, \ldots$

a) Find the product of the first two terms, then the first three terms, the first four terms, and the first five terms.
b) What is the product of the first 99 terms of this sequence?
c) How many consecutive terms, beginning with the first term, must be multiplied to get a product of $1/2001$?

Answer: a) $1/3, 1/4, 1/5, 1/6$
b) $1/100$
c) 2000

Example: A sequence is given as $1, 1/2, 1/4, 1/8, 1/16, 1/32, \ldots$, What is the quotient of the 58th term divided by the 59th term?

Answer: 2

Although the students should know how to use the basic properties applied to operations with fractions, a formal study of these properties will be delayed until grade 8. The following *properties* are true for the set of rational numbers:

---

(1) Commutative Property of Addition: $a+b = b+a$
(2) Commutative Property of Multiplication: $a \cdot b = b \cdot a$
(3) Associative Property of Addition: $(a+b)+c = a+(b+c)$
(4) Associative Property of Multiplication: $(a \cdot b) \cdot c = a \cdot (b \cdot c)$
(5) Distributive Property of Multiplication over Addition: $a \cdot (b+c) = (a \cdot b) + (a \cdot c)$
(6) Identity element for multiplication is 1.
(7) Every rational number except 0 has a multiplicative inverse or reciprocal. The reciprocal of a/b is b/a, where $a \neq 0$ and $b \neq 0$.
(8) Identity element for addition is 0.
(9) Every rational number, a, has an additive inverse, $-a$.

---

c. Order of operations

The order of operations (outlined under II-A-9 for whole numbers) can now be extended to include answers that are positive rational numbers.

Example: Are the expressions below equal in value? If not, which is larger?

a.	$3 \div 2 - 4$	b.	$3 \div (2 \div 4)$
	$3/2 \div 4$		$3 \div 1/2$
	$3/2 \times 1/4$		$3 \times 2/1$
	$3/8$		6

Answer: $3 \div (2 \div 4)$ names the larger value.

4. Converting Common Fractions to Decimal Fractions

A *decimal*, or a decimal fraction, is a fraction whose denominator is understood to be a power of 10 and is designated by the position of a decimal point. For example, $3/10$ is .3 or 0.3, and $7/100$ is .07 or 0.07.

Every common fraction (in effect, every rational number) can be converted to a decimal by dividing the numerator of the fraction by its denominator. Since $3/10 = 3 \div 10$, and since $3 = 3.0$, it is possible to show $3/10$ as:

$$10\overline{)3.0}\quad .3$$

a. Terminating decimals

Every rational number can be written as either a terminating decimal or an infinitely repeating decimal. In the changing of a fraction to a decimal, a terminating decimal occurs when a remainder of 0 is reached in the division process, as in:

$3/4 = .75$ $\quad 1/8 = .125$ $\quad 17/20 = .85$

b. Repeating decimals

A common fraction is an infinitely repeating decimal when a remainder of 0 is not reached, and the digits in the quotient repeat in a definite pattern. A repetend, indicated as a bar over the repeating digits, is used in writing the repeating decimal.

$1/3 = 0.333... = 0.\overline{3}$ $\quad 1/30 = 0.0333... = 0.0\overline{3}$

$1/33 = 0.030303... = 0.\overline{03}$ $\quad 1/6 = 0.1666... = 0.1\overline{6}$

$1/7 = 0.142857142857... = 0.\overline{142857}$

A terminating decimal is actually a repeating decimal in which 0 repeats. For example, $3/4 = .75000... = .75\overline{0}$. Therefore, every rational number can be expressed as a repeating decimal.

A *unit fraction* is a common fraction with a numerator of 1. Multiples of unit fractions, and their decimal equivalents, offer many interesting patterns for study.

$1/9 = .111... = .\overline{1}$ $\quad 4/9 = .444... = .\overline{4}$ $\quad 7/9 = .777... = .\overline{7}$

$2/9 = .222... = .\overline{2}$ $\quad 5/9 = .555... = .\overline{5}$ $\quad 8/9 = .888... = .\overline{8}$

$3/9 = .333... = .\overline{3}$ $\quad 6/9 = .666... = .\overline{6}$ $\quad 9/9 = .999... = .\overline{9}$

The students realize that $9/9 = 1$. A lively discussion is guaranteed when they are shown that $.9999\ldots = .\overline{9} = 1$.

Optional: Since computers and hand calculators work only with numbers in decimal notation, some interesting activities can be introduced in which fractions and mixed numbers are entered into these devices. The limitation of the display and the resulting rounding errors on these devices show the need for knowing correct conversions from fractions to decimals.

The method by which a number, such as $3\frac{5}{8}$, is entered on a calculator will vary from one device to another. A student may realize that $3\frac{5}{8} = 3 + \frac{5}{8}$, but an entry of $3 + 5 \div 8$ could equal 1. To force the correct order of operations, it *may* be necessary to enter the number as follows:

$5 \div 8 + 3 = 3.625$

Calculators and computers can be useful in studying problems derived from sequences.

Example: The Fibonacci sequence is 1, 1, 2, 3, 5, 8, 13, 21, 34,... Use a calculator or a computer to divide each term by its succeeding term, and express these quotients in decimal form: $1/1$, $1/2$, $2/3$, $3/5$, $5/8$, $8/13$, $13/21$, $21/34$, $34/55$,...

Carry the process out to twelve terms, or more. Check the library to see how this pattern is related to the "Golden Ratio," or the "Golden Section."

5. Converting Decimal Fractions to Common Fractions

If a student knows how to read a *terminating decimal* using place value, then the student may immediately write the number as a common fraction. For example, 0.15 is read as "15 hundredths." Read the decimal; write the fraction; reduce it.

$0.15 = 15/100 = 3/20$

The *repeating decimals* $0.333... = 0.\overline{3}$ and $0.666... = 0.\overline{6}$ should be instantly recognized as common fractions, namely $1/3$ and $2/3$.

6. Operations and Problem Solving with Decimals

a. Four basic operations

The four basic operations with positive decimals, and zero, should be reviewed through problem-solving situations. A comparison to common fractions may help to stress the proper placement of the decimal point in the corresponding decimal solution.

(a) $.7 + .03$ or $.70 + .03 = .73$

$7/10 + 3/100 = 70/100 + 3/100 = 73/100$

(b) $.4 \times .14 = .056$

$4/10 \times 14/100 = 56/1000$

(c) $.9\overline{).63} = 9\overline{)6.3}$ $\quad .7$

$63/100 \div 9/10 = 63/100 \times 10/9 = 7/10$

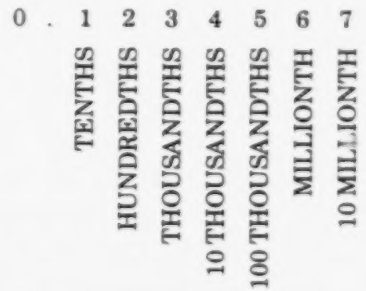
b. Estimation

Estimation of answers to decimal problems should be discussed. Consider how one simple method involving the use of the "leading digit" is used to find estimates of the three problems just solved:

(a) .7 + .0 gives an estimated answer of .7
(b) .4 × .1 gives an estimated answer of .04
(c) .6 ÷ .9, or 6 ÷ 9, gives an estimate of $\frac{2}{3}$

c. Rounding decimals

In grade 6, the students were taught to round decimals only to the nearest tenth and to the nearest hundredth. These limitations probably fit most problem-solving situations. Because of the use of computers and calculators, however, rounding should be extended in grade 7 to six decimal places (the nearest millionth) or to seven decimal places. The students should have some practice in *reading decimals* to a corresponding number of places.



There are many procedures used to round decimals. At a minimum, the students should be exposed to two:

(1) The rule of "5 or more." For example, $\frac{4}{11} = 0.\overline{36} = 0.36\ 36\ 36\ldots$ is rounded to three decimal places as 0.364, and rounded to two decimal places as $0.\overline{36}$.
(2) The rule of "rounding all values upward," as in shopping. This rule is more easily understood by making some reference to money. For example, $\frac{4}{11} = 0.36\ 36\ 36\ldots$ is rounded in shopping "to the nearest cent" as 0.37, clearly rounding upward.

Optional: At this point, the teacher may wish to discuss the concept of *truncating* numbers, best shown by using certain hand calculators. For example, $\frac{4}{11}$, or 4 ÷ 11, is displayed on some calculators as 0.3636363, a *truncated number* or a number that has been "cut off" after a certain number of decimal places.

Optional: *Exponents* and *principal square roots* (restricted to rational numbers which are perfect squares) provide practice in decimal computation within a new setting.

$(.4)^2 = .064 \qquad \sqrt{2.25} = 1.5$

Example: Mr. McCormack has in his pocket exactly one of each type of U.S. coin up through a silver dollar. What is the total amount of these coins?

Answer: $1.91

Example: An object weighs 6 times as much on earth as on the moon.

a) A rock weighs 3.54 kg on the moon. Find its weight on the earth.
b) An astronaut weighs 154 kg on earth. Find to the nearest tenth of a kilogram the weight of the astronaut on the moon.

Answers: a) 21.24 kg b) 25.7 kg

Example: The Explorers Club has $20 in its treasury. How many boxes of donuts, each containing a dozen, can the club purchase if donuts cost $2.59 per dozen?

Answer: 7

Optional: Sequences involving decimals may also be studied to reenforce operations and to study patterns.

Example: Each succeeding term in the sequence 1, .2, .04,. . . is found by multiplying .2 and the previous term.

a) Write the first eight terms of this sequence.
b) A new sequence is found by subtracting terms in the first sequence, namely: 1st term − 2nd term; 2nd term − 3rd term; 3rd term − 4th term; and so on. Write the first seven terms of this new sequence.

Answer: a) 1, .2, .04, .008, .0016, .00032, .000064, .0000128
b) .8, .16, .032, .0064, .00128, .000256, .0000512

7. Scientific Notation for Numbers Larger Than or Equal to One

In *scientific notation*, a number is expressed as a product of two quantities: the first is a number equal to or greater than 1, but less than 10; the second is an integral power of 10.

$30\,000 = 3 \times 10\,000 = 3 \times 10^4$

$195\,000 = 1.95 \times 100\,000 = 1.95 \times 10^5$

$7 = 7 \times 1 = 7 \times 10^0$

Many scientific measures are stated by using estimates, or rounded numbers, of very large or very small quantities. Scientific notation is used to write such numbers. In grade 7, the students should learn this notation for large numbers ($n \geq 1$). Scientific notation for small numbers ($0 \leq n < 1$) will be studied in grade 8.

Practical examples, such as distances to planets or

populations of countries, appear in many commonly used reference books. The students should be able to convert numbers from scientific notation to decimal notation, as well as from decimal notation to scientific notation. Operations involving numbers written in scientific notation are *not* recommended at this level.

However, the students may see numbers displayed in scientific notation on calculators or in computer output. As an exercise using such devices, have the students enter a number, say 2.7, and use repeated multiplication by 10 to write the first fifteen terms of a sequence: 2.7, 27, 270, 2700,. . .

Is the fifteenth term 2.7×10^{14}? How does the device show this number?

8. Inequalities

a. Comparisons of rational numbers

Two rational numbers can be compared to see if the first is greater than (>), less than (<), or equal to (=) the second. This comparison of rational numbers can be shown by various methods:

(1) Convert the numbers to common fractions with like denominators.

$$7/10 \; ? \; 3/4$$

$$7/10 \times 2/2 \; ? \; 3/4 \times 5/5$$

Since $14/20 < 15/20$, then $7/10 < 3/4$

(2) Convert the numbers to decimals.

$$2/3 \; ? \; 5/8$$

$$0.\overline{6} \; ? \; 0.625$$

Since $0.666... > 0.625$, then $2/3 > 5/8$

In effect, the set of rational numbers can be *ordered*. On a standard horizontal number line, the greater of two rational numbers is positioned to the right of the smaller.

Example: Find all rational numbers of the form a/b where a and b are both single digit prime numbers, and a/b is greater than 1. Write these numbers in order from the smallest to the largest.

Answer: $7/5, 3/2, 5/3, 7/3, 5/2, 7/2$

b. Number line graphs

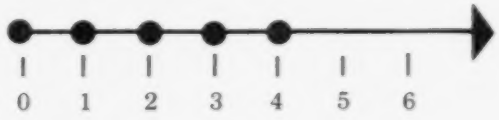
Number line graphs, used to display sets of numbers, are frequently associated with inequalities. In grades K-6, simple conditions of inequality such as "is greater than" and "is less than" were covered. Compound conditions of inequality should now be formally introduced:

"is greater than or equal to"	$\geq$
"is less than or equal to"	$\leq$

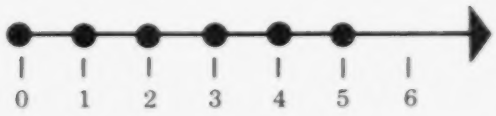
The students should graph sets of numbers that are solutions for inequalities, and compare graphs of whole numbers with graphs of positive rational numbers and zero. Consider the following examples:

WHOLE NUMBERS

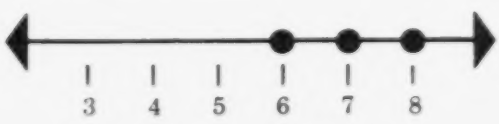
(1) Numbers less than 5



(2) Numbers less than or equal to 5

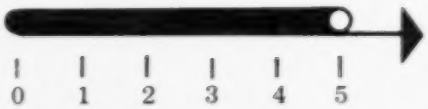


(3) Numbers greater than 5



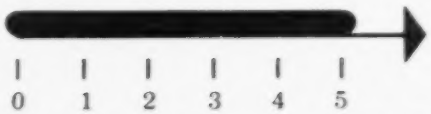
POSITIVE RATIONAL NUMBERS AND ZERO

(1) Numbers less than 5



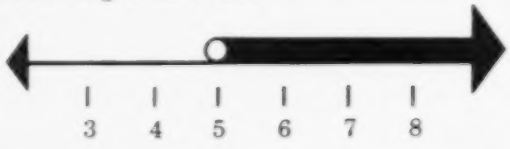
The empty circle at 5 indicates that 5 is *not* in the graph.

(2) Numbers less than or equal to 5



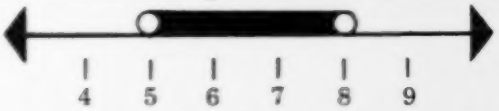
The darkened circle at 5 indicates that 5 is in the graph.

(3) Numbers greater than 5



The concept of betweenness may be discussed informally at this stage and applied to graphing of sets of numbers:

(4) Rational numbers greater than 5 and less than 8.



Note that graphs of the rational numbers are shown on the real number line. Despite the density of the set of rational numbers, there are gaps or holes in these graphs since some real numbers are not rational. This discussion, however, is best postponed until grade 8 when real numbers are studied.

c. Problem solving

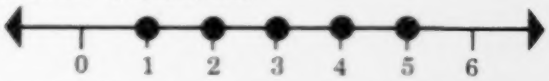
Problem-solving situations involving simple inequalities should be presented to the students. Solutions may be given in words, shown on a graph, or (where possible) listed. The students should understand that those solutions which contain an infinite number of rational numbers cannot be listed.

Phrases such as "at least" and "at most" may appear in some problems, depending upon the ability of the students:

"At least 3" means "3 or more."
"At most 4" means "4 or less."

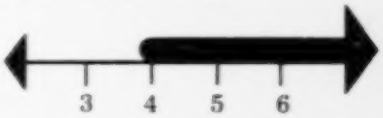
Example: Mrs. Levine's car seats at most 5 people. How many people can be seated in this car while it is being driven?

Answer: 1, 2, 3, 4, 5 or,



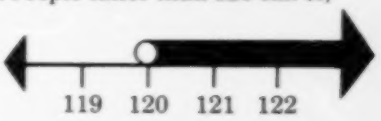
Example: Mary Irene has been sleeping for at least 4 hours. How many hours has she been sleeping?

Answer: 4 hours or more. or,



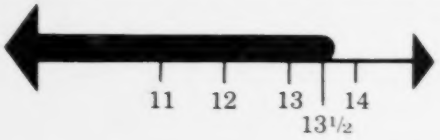
Example: "To get on this amusement ride, you must be taller than the line shown here." If the line is 120 centimeters off the ground, who can get on the ride?

Answer: People taller than 120 cm. or,



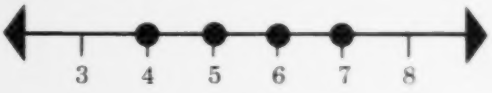
Example: A sign at a tunnel states that the clearance is 13′6″. Find the heights of all vehicles that can safely pass through the tunnel.

Answer: Heights of 13′6″ (or 13½ feet), or less. or,



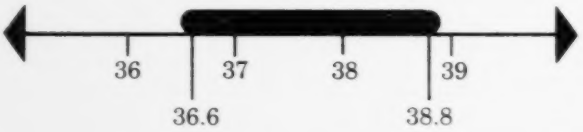
Example: Mr. Gehringer sends between 3 and 8 students to the chalkboard every day to solve math problems. How many students does he send daily?

Answer: 4, 5, 6, 7 or,



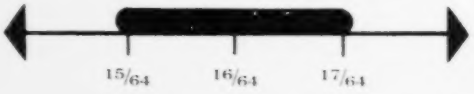
Example: The normal body temperature for a person is 37° C. During an illness, Mr. Phillips' temperature dropped 0.4° C below normal, and rose 1.8° C above normal. What temperatures did Mr. Phillips have during his illness?

Answer: 36.6° C or more, and 38.8° C or less. or, At least 36.6° C and at most 38.8° C. or, Between 36.6° C and 38.8° C, *inclusive*. or,



Example: A machine is set to make hamburgers that weigh ¼ pound. However, the weights sometimes vary by as much as 1/64 pound. Find all possible weights of the hamburgers made by this machine.

Answer: At least 15/64 pound and at most 17/64 pound. or, Between 15/64 pound and 17/64 pound, *inclusive*.



III. RATIO, PROPORTION, AND PERCENT

The K-6 *Syllabus* gives an extensive treatment of ratio; proportion and percent are first introduced in grade 6. The work described below for grade 7 builds upon that foundation, and is to be a major area of study.

Herein are many opportunities for the students to use the problem-solving strategies presented earlier. Furthermore, the nature of the computational skills required to solve problems offers yet another vehicle for *diagnosis and practice of these skills.*

A. RATIO

1. Definition and Forms of Notation

A ratio is a comparison of two numbers by division. The ratio of a to b, where $b \neq 0$, is the quotient a/b. For example, the ratio of 3 to 6 is $3/6$, or $1/2$ in reduced form.

The students should be familiar with the most common forms of notation for a ratio. These forms, each read as "4 to 1," are written as follows:

4 to 1 4:1 $4/1$

Other statements, such as "4 is to 1" and "4 divided by 1," also express a ratio. Although the number 4 (with a denominator of 1 understood) is an acceptable form of a ratio, this notation does not clearly display a comparison of two numbers and should be avoided at this level.

When comparing numbers resulting from measurements, the terms of the ratio must be expressed in the same units of measure to show the ratio as a number.

Example: Find the ratio of 33 millimeters to 2 centimeters.

$$33\text{mm}/2\text{cm} = 33\text{mm}/20\text{mm} = 33/20$$

Answer: $33/20$, or 33:20, or 33 to 20.

(The teacher may wish to review systems of measurement here, rather than in sections VII-B and C of this *Syllabus.*)

Practice should include exercises like:

a) Find the ratio of 6 kg to 11 kg (6:11)
b) Find the ratio of 4 days to 1 week (4:7)
c) Find the ratio of 150 m to 1 km (150:1000 or $15/100$)
d) Find the ratio of 3" to 1' (3:12 or $1/4$)

2. Rates as Ratios

In the comparing of two numbers that result from measurements which cannot be expressed in the same units, the ratio is called a *rate*. For example, if a car travels 80 km in 2 hours, its average rate of speed is 80 km/2 hr. , or 40 km/hr. This rate is read as "40 kilometers per hour."

Examples should include rates like:

a) 3 pens for 80¢
b) 55 miles per hour, written 55 mph
c) 4 marking periods per year
d) 30 characters per second
e) 4 tbsp. lemonade crystals to 1 qt. water

Note: Although the measures in the last rate may be expressed in the same units, such ratios are cumbersome and/or meaningless.

3. Problem Solving

Many applications of both rates and ratios should be studied. Budgets, surveys, and local geographic matters are sources of examples. *Student experiments* can make the topic of ratio come alive. For example:

a) In nature there are many patterns – including the size and shape of the human body. Pair the students off and give each pair a meter-long string. Have the students measure and record as follows:

1. Measure the distance around the base of the thumb, using the string.
2. Double that length. It should fit around your wrist.
3. Double that length. It should fit around your neck.
4. Double that length. (Here's the hard one!) It should fit around your waist.

b) Show that the relationships just studied are ratios. What are the ratios for the various measures?

neck to thumb (4:1, 4 to 1)
wrist to thumb (2:1, 2 to 1)
neck to wrist (2:1)
waist to thumb (8:1)

c) Use a tape measure to compare your height to the length of your foot. Change each ratio to a decimal (compared to 1), correct to the nearest tenth.

	Height	Length of Foot	Ratio
Elsie:	152 cm	25 cm $^{152}/_{25}$	6.1 to 1
Bob:	144 cm	23 cm $^{144}/_{23}$	6.3 to 1
Teacher:	180 cm	30 cm $^{180}/_{30}$	6.0 to 1

Compare the ratios found by the students in the class. Most ratios will be close to "6 to 1," another pattern in nature.

Tangrams, geoboards, and the measurement of objects such as boxes and tin cans offer other experimental examples for the study of ratio.

Example: Ellen spent $3 for movies, $2 for magazines, $1 for sodas, and $1 for odds and ends. Find the ratio of the amount spent for movies in relation to the total amount.

Answer: 3:7

Example: How many boys are in our class? how many girls? What is the ratio of boys to girls? of girls to the whole class? of the class to the boys?

Example: How many voters in our county are registered Democrats? registered Republicans? others? Find various ratios.

B. PROPORTION

1. Definition and Forms of Notation

A *proportion* is a statement that two ratios are equal. To show that the ratios "4 to 8" and "1 to 2" are equal, we write:

$^4/_8 = {}^1/_2$ or 4:8 = 1:2

Each proportion shown above is read "4 is to 8 as 1 is to 2."

a. Cross-products rule

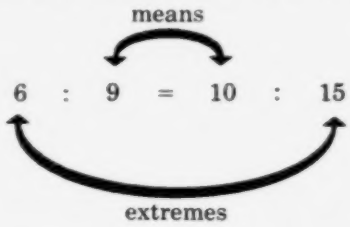
In grade 6, the students learned the cross-products rule, namely, if the cross products of two ratios (or two fractions) are equal, then the terms are in proportion. For example:

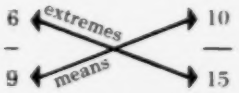
$^3/_4 = {}^6/_8$ is a proportion because

$$\frac{3}{4} \times \frac{6}{8} \qquad 3 \times 8 = 4 \times 6$$
$$24 = 24$$

b. Means and extremes

The students should now be introduced to an alternate rule. That is, when a proportion is written in a horizontal format, the first and fourth terms are called the extremes and the second and third terms are called the means of the proportion. These concepts can be applied to proportions expressed in fractional notation.





Using either notation, $9 \times 10 = 6 \times 15$, or $90 = 90$, illustrates that: *In a proportion, the product of the means is equal to the product of the extremes.*

If the product of the means is not equal to the product of the extremes, or if the cross products of two ratios are not equal, then the terms are not in proportion.

The students should understand that for any given ratio there is an infinite number of equal ratios as in $^1/_3$, $^2/_6$, $^3/_9$, $^4/_{12}$..., and that any two will form a proportion as in $^2/_6 = {}^3/_9$. The proof of the cross-products rule is *optional.*

Example: Which two of the following ratios form a proportion?

3:5 6:12 9:15 4:10

Answer: 3:5 = 9:15

Example: The dimensions of a snapshot are 5 cm by 8 cm. A larger photograph has dimensions of 8 cm by 12 cm. Are the dimensions of the photographs in proportion?

Answer: No

2. Solving Proportions

If any three of the four terms in a proportion are given, the fourth term can be determined. A variable is used to

represent this fourth term and, in most cases, its value is found by solving an equation.

Example: 4:14 = 6: W

$4 \times W = 14 \times 6$

$4W = 84$

Answer: W = 21

Example: $^{10}/_{N} = {}^{25}/_{15}$

$25 \times N = 10 \times 15$

$25N = 150$

Answer: N = 6

In some cases, a student may instantly recognize the missing term because of his/her knowledge of equivalent fractions. Although the answer may be accepted without the solution of an equation, the teacher may require that the student perform a check of the proportion as a safeguard. In the following example, a student knows that $^{2}/_{3}$ and $^{4}/_{6}$ are equivalent fractions.

Example: Solve for n

2:3 = n:6

Answer: n = 4

Check: 2:3 = 4:6

3•4 = 2•6

12 = 12

Optional: Alternate methods, such as reducing to lower terms and increasing to higher terms, may be used to solve proportions.

3. Problem Solving

Simple problems involving enlargements, reductions, scale models, comparisons, and rates should be studied to give the students practice in writing proportions as well as solving them. Activities could include the actual drawing of a scale model such as a floor plan.

Example: A photograph is enlarged so that each of its dimensions is 4 times its original size. If Carlos measures 3 cm in the original photograph, how tall will his image be in the enlargement?

Answer: 12 cm

Example: A map has a scale of 1 inch = 5 miles. Two cities are 3 inches apart on the map. How far apart are they in real life?

Answer: 15 miles

Example: To save money, a gasoline company reduces the size of its maps to $^{3}/_{5}$ of their original dimensions. If two cities are 15 cm apart on the original map, how far apart are they on the new map?

Answer: 9 cm

Example: A model of the Concorde is built to a scale of 1:100. If the real Concorde is 59 m long, how long is the model?

Answer: 0.59 m, or 59 cm

Example: If 4 records cost $31, how much should 3 records cost?

Answer: $23.25

Example: If a dozen eggs are needed to make a gallon of pancake batter, how many eggs are needed for 3 quarts of the batter? (The students should know a dozen is 12, and 1 gallon is 4 quarts.)

Answer: 9

C. PERCENT

1. Definition

A *percent* is a ratio with a denominator of 100. Percent means "per hundred" or "hundredths," and is represented by the symbol, %. For example $27\% = {}^{27}/_{100} = .27$, and $4\% = {}^{4}/_{100} = .04$.

Percents are often used instead of decimals or common fractions because all percents have a common denominator of 100 for easy comparison. Consider the following problem.

Example: The best player on the girls' basketball team made 12 baskets in 20 tries. The best player on the boys' basketball team made 14 baskets in 25 tries. Who was the better shot?

Girl: $^{12}/_{20}$ Boy: $^{14}/_{25}$

To compare these results, find equivalent fractions that have like denominators. Since the LCD (least common denominator) of 20 and 25 is 100, the results can be written as percents.

Girl: $^{12}/_{20} = {}^{60}/_{100} = 60\%$

Boy: $^{14}/_{25} = {}^{56}/_{100} = 56\%$

Answer: The girl is the better shot.

2. Decimal/Fraction/Percent Equivalents

There are many ways to write numerals that show a percent. For example, two percent may be written as: 2%; 0.02; $^{2}/_{100}$; $^{1}/_{50}$; 2 ÷ 100; 2:100; and so forth.

A ratio, such as 4 to 25, may be written as $^{4}/_{25}$, which is equivalent to $^{16}/_{100}$. Since $^{16}/_{100} = 16\%$, and $^{16}/_{100} = 0.16$, the students should be able to identify and use various methods to find decimal-fraction-percent equivalents. Most of the equivalents studied in this unit should

involve whole number percents. However, depending upon the ability of the class, percents involving mixed numbers and decimals (such as $7\frac{1}{2}\%$ and 12.25%) may be presented.

To change a decimal to a percent, move the decimal point two places to the right and annex the % symbol. *To change a percent to a decimal*, move the decimal point two places to the left and drop the % symbol.

$.85 = 85\%$ $\quad 12\% = .12$

$.03 = 3\%$ $\quad 33\frac{1}{3}\% = .33\frac{1}{3}$ or $.\overline{3}$

$.625 = 62.5\% = 62\frac{1}{2}\%$ $\quad 2\frac{1}{2}\% = .02\frac{1}{2}$ or .025

$.1225 = 12.25\% = 12\frac{1}{4}\%$

To change a common fraction to a percent, use any of several acceptable procedures:

a) Find an equivalent fraction with a denominator of 100, then write the percent.

$$\frac{3}{5} = \frac{3}{5} \times \frac{20}{20} = \frac{60}{100} = 60\%$$

b) Change the fraction to a decimal, and the decimal to a percent.

$\frac{3}{5} = 0.60 = 60\%$ $\quad \frac{1}{12} = .08\overline{3} = .08\frac{1}{3} = 8\frac{1}{3}\%$

c) Since 1 = 1.00 = 100%, multiply by 100%.

$\frac{3}{5} = \frac{3}{5} \times 100\% = 60\%$

$\frac{1}{12} = \frac{1}{12} \times 100\% = \frac{25}{3}\% = 8\frac{1}{3}\%$

To change a percent to a common fraction, use any of several acceptable procedures:

a) Write the percent as a fraction with a denominator of 100, and reduce it if possible. (This method may lead to complex fractions which should be simplified.)

$35\% = \frac{35}{100} = \frac{7}{20}$ $\quad 9\% = \frac{9}{100}$

$$7\frac{1}{2}\% = \frac{7\frac{1}{2}}{100} = \frac{7\frac{1}{2}}{100} \times \frac{2}{2} = \frac{15}{200} = \frac{3}{40}$$

b) Change the percent to a decimal, and the decimal to a fraction.

$40\% = .40 = .4 = \frac{4}{10} = \frac{2}{5}$

$12\frac{1}{2}\% = 12.5\% = .125 = \frac{125}{1000} = \frac{1}{8}$

Use any mathematically correct procedure to find decimal-fraction-percent equivalents. Percents greater than 100% and less than 1% are *optional* at this level, but will be studied in grade 8.

3. Working with Percents

The students should be taught to solve problems with percents, but no mention should be made to the students of three cases of percents per se. The use of proportions provides a universal method for solution and should be presented. However, as the students discover alternate methods for solution, these procedures should be allowed.

The students should understand that in writing a proportion one ratio consists of two numbers, and the second ratio is a percent expressed as a fraction.

a. Finding a percent of a number

Example:

Find 30% of 70 or

WRITE $\frac{N}{70} = \frac{30}{100}$	WRITE $\frac{N}{70} = \frac{3}{10}$
SOLVE $100N = 2100$	$10N = 210$
$N = 21$	$N = 21$

Answer: 21

The students may realize that 30% of 70 is equivalent to a multiplication problem, and use this more direct approach:

$.30 \times 70 = 21.00 = 21$ or $\frac{3}{10} \times \frac{70}{1} = 21$

(The teacher who wishes to solve percent problems by equations may integrate the writing of equations from the algebra strand within this unit. Another approach is to teach the algebraic technique later as an alternate method to reenforce percent.)

b. Finding what percent one number is of another

Example: What percent of 12 is 9?

WRITE $\frac{9}{12} = \frac{x}{100}$ $\quad$ since $x = 75$

SOLVE $12x = 900$ $\quad \frac{75}{100} = 75\%$

$x = 75$

Answer: 75%

Since the two given numbers form a ratio or a fraction, the students may realize that this problem is equivalent to changing a fraction to a percent, and work accordingly:

$\frac{9}{12} = \frac{3}{4} = \frac{75}{100} = 75\%$ or $\frac{9}{12} = \frac{3}{4} = 75\%$

c. Finding a number when a percent of it is given.

Example: If 15% of a number is 6, what is the number?

WRITE $\frac{6}{N} = \frac{15}{100}$ or $\frac{6}{N} = \frac{3}{20}$

SOLVE 15N = 600	3N = 120
N = 40	N = 40

Answer: 40

The students may use problem-solving techniques to find an alternate method for solution. For example:

15% is 6	DIVIDE by 3
5% is 2	MULTIPLY by 20
100% is 40	

4. Problem Solving

Both standard problems (commission; discount; taxes; simple interest) and nonstandard percent problems should be solved. Percent of change, such as percent of increase/decrease, is optional in grade 7 but is required in grade 8.

Each percent problem should be translated to a simpler format and solved.

Example: Mr. Ruiz earns a 6% commission for every house he sells as a real estate agent. Find his commission on a house that sold for $71,000.

(Find 6% of $71,000.)

Answer: $4,260

Example: A $40 tape recorder is placed on sale at 20% off. Find: (a) the discount (b) the sale price of the recorder.

(a. Find 20% of $40. b. Subtract the discount from the original price, or find the sale price directly by 80% of $40.)

Answer: (a) $8 (b) $32

Example: At a sale where all items were marked 25% off, Barbara bought a pair of jeans for $21. What was the original selling price of the jeans?

(Find the number where 75% of it is $21.)

Answer: $28

Example: A record is on sale for $6.95. (a) If the rate of tax is 7%, find, correct to the nearest cent, the sales tax on the record. (b) Find the total cost of the purchase.

(Find 7% of $6.95)

Answer: (a) $.49 (b) $7.44

Interest has become so complex in today's world that problems involving simple interest have limited use unless the simple interest is regarded as an "estimate" of the actual interest.

In the rule I=PRT, the students should understand that I=interest, an amount of money earned or charged; P=principal, an amount used as a base; R = annual rate, which is usually a percent; and T=time in years. Thus, 6 months is treated as $\frac{1}{2}$ year, and 3 months as $\frac{1}{4}$ year.

Example: To pay for a car, Bob borrowed $4,500 at 16% per year for 2 years. How much interest must he pay on this loan?

(I = $4,500. × 16% × 2)

Answer: $1,440

Example: The sale price of a pair of stereo speakers at 20% off is $98. The original price is listed at $115. Do you really get 20% off?

(Find 80% of $115.)

Answer: The sale price at 20% off should be *$92*, not *$98*.

Example: Sheila gave her brother 12 coins from her collection of 80 coins. What percent of her collection did she give him?

(12 is what percent of 80?)

Answer: 15%

Example: Patrick's snack contained 186 calories, which was 6% of his daily requirement. What is his daily calorie requirement?

(186 is 6% of what number?)

Answer: 3,100 calories

Example: On a math test given to 30 students, the following grades were earned:

3 A's; 8 B's; 12 C's; 5 D's; 2 F's

(a) What percent of the class got *each* of the following grades, rounded to the nearest whole percent: A,B,C,D,F?

(b) Is the sum of the five percents found in part (a) equal to 100%? Why?

Answer: (a) 10% A; 27% B; 40% C; 17% D; 7% F

(b) No, the sum is 101%, due to rounding.

IV. PROBABILITY AND STATISTICS

A major strand in the K-6 *Syllabus* is the study of probability and statistics. In those grades, probability was introduced from an empirical approach and briefly extended to the theoretical definition. The students were exposed to tree diagrams, sample spaces, and sampling procedures. Counting of arrangements (permutations) and groups (combinations), along with a discussion of the multiplication principle, was introduced. In essence, every concept of probability to be studied in grade 7 has been seen in earlier grades, but most likely in a more informal setting.

Experiments and activities, still a part of probability in grade 7, are now tempered by the beginnings of a formal structure that includes definitions and rules. Terminology new to the students will be so noted. A set theory approach to probability (not taken in the K-6 *Syllabus*) will be stated within this curriculum. This approach, however, is not mandated; the teacher may present probability without the use of sets. In grades K-6, in the area of statistics, the students learned to collect and organize data, determine the mean (average) and range, and construct various graphs (bar, line, picture, circle, and double bar). In grade 7, the median and the mode will be introduced. Frequency distributions with intervals of equal length will lead to the construction of histograms, another new topic.

A. PROBABILITY

1. Empirical vs. Theoretical Probability

In grades K-6, the students performed experiments to make predictions, but the term "'empirical probability" was never used. A rule for theoretical probability was given.

Discuss the difference between empirical and theoretical probability.

Empirical probability is an estimate or a guess of an event's happening, based upon gathered information and the ratio:

$$\frac{\text{The number of times the event happens}}{\text{The total number of trials}}$$

For example, if Sam tosses a coin 20 times and gets 9 heads, he may guess that the probability of heads is $^9/_{20}$. If Karen tosses a coin 20 times and gets 12 heads, she may guess that the probability of heads is $^{12}/_{20}$.

Theoretical probability is given by the rule, $P(E) = {}^f/_n$, or:

$$\text{The probability of an event} = \frac{\textit{The number of favorable outcomes}}{\text{Total number of possible outcomes}}$$

For example, there are two possible outcomes when a coin is tossed, namely heads or tails, written H or T. In stating the probability for a head, there is only one favorable outcome. So, $P(H) = {}^1/_2$, or the theoretical probability of obtaining a head is $^1/_2$.

Why should we bother with empirical probability if "incorrect" estimates appear to be given? Let us refine the definition of empirical probability.

2. Empirical Probability

In K-6, the students were taught that an increased number of trials in an experiment produced more reliable results. The students were not taught, however, any of the following terms: relative frequency; cumulative results; cumulative relative frequency.

In grade 7, student experiments should be performed to refine the definition of empirical probability. Start with a *fair object* so that the students will sense that the empirical approach taken verifies the theoretical probability learned in earlier grades. *Fair objects* (coins, dice, cards, etc.) are those in which each different result has an equal chance of happening. Consider this example:

Experiment 1: Each student is to toss a coin 20 times and record the results. Empirically determine the probability of a head.

a. Cumulative relative frequency

Perhaps Sam gets 9 heads out of 20 tosses; the fraction $^9/_{20}$ is called the relative frequency of this event. If Karen gets 12 heads out of 20 tosses, then the *relative frequency* for her is $^{12}/_{20}$. Neither relative frequency is equal to the probability $^1/_2$, but what happens if we combine results?

The *cumulative* (or total) number of heads is 9 + 12, or 21; the cumulative number of tosses is 20 + 20, or 40; and the cumulative relative frequency is $^{21}/_{40}$.

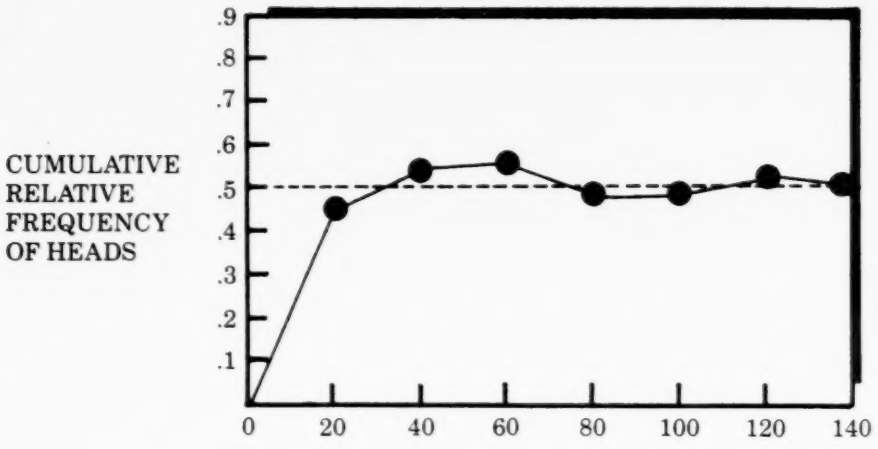
Continue this process and organize student results in a table. Find the cumulative relative frequency for each additional 20 tosses. To see how close each new result is to $^1/_2$ or .5, express the fraction as a decimal to the nearest thousandth.

	Number of Heads	Number of Tosses	Cumulative Number of Heads	Cumulative Number of Tosses	Cumulative Relative Frequency
Sam	9	20	9	20	$^9/_{20} = .450$
Karen	12	20	21	40	$^{21}/_{40} = .525$
Ronnie	12	20	33	60	$^{33}/_{60} = .550$
David	6	20	39	80	$^{39}/_{80} = .488$
Marcia	10	20	49	100	$^{49}/_{100} = .490$
Gary	13	20	62	120	$^{62}/_{120} = .517$
Carissa	9	20	71	140	$^{71}/_{140} = .507$

For these students, the relative frequencies varied greatly, from $^6/_{20}$ to $^{13}/_{20}$, with only one student at $^{10}/_{20}$. However, the cumulative relative frequency is $^{71}/_{140}$, a number very close to $^1/_2$.

The students should construct a *graph* of the results to see that, as more trials are performed, the cumulative relative frequency "approaches" or "levels out" at $^1/_2$.

CUMULATIVE RELATIVE FREQUENCY
OF HEADS FOR 0 TO 140 TOSSES



As more data are gathered, the assignment of a probability by empirical methods becomes more clear. When the line graph "levels out" at some number, or the cumulative relative frequency is "stabilized" at that number, then the number is assigned as the empirical probability of the event. Notice that $\frac{1}{2}$, not $\frac{71}{140}$, is assigned as the probability for heads because it is felt that $\frac{1}{2}$ makes more sense. A definition is now in order:

Empirical probability is the best possible estimate or guess of an event's happening, based upon the cumulative relative frequency of the event.

Other experiments involving fair objects or events may be performed in place of, or in addition to, the tossing of a coin. For example:

Experiment 2: Each student tosses a fair 6-sided die 30 times. Empirically find the probability of rolling a "4."

Answer: P(4) = $\frac{1}{6}$

Experiment 3: Each student opens a telephone book to any page and looks at the first 30 telephone numbers given. Using empirical methods, find the probability that a telephone number ends in a "7."

Answer: P(7) = $\frac{1}{10}$

Experiment 4: Each student shuffles a deck of 52 playing cards, selects a card without looking, and then records its suit. Do this 20 times each, and empirically find the probability that a "heart" is selected.

Answer: P(heart) = $\frac{1}{4}$

b. Biased objects

Perform at least one experiment with a biased object to show the power of an empirical approach in cases where theoretical rules do not apply. Biased or unfair objects (weighted coins, shaved dice, etc.) are those in which each different result does *not* have an equal chance of happening. Consider these examples:

Experiment 5: Each student tosses the same shaped paper cup 25 times. The cup can land in one of three positions:

Find P(side), the probability that the cup lands on its side.

Experiment 6: Each student drops a thumbtack 20 times on a hard surface. The tack can land in one of two ways.

Find P(thumbtack lands point up).

Every probability may be determined by empirical procedures. Only certain probabilities, such as those involving fair objects, can be found by the theoretical rule.

Example: In which of the following situations *must*

empirical methods be used to determine the probability?

(1) A spinner is divided into 4 equal parts. What is the probability that the arrow will stop on 3? (See Figure 1)

(2) A spinner is divided into 4 unequal parts. What is the probability that the arrow will land on 3? (See Figure 2)

FIGURE 1 FIGURE 2

(3) What is the probability that a student in your class rides to and from school on a bus?

(4) What is the probability that a woman will live to be 74 years of age or older?

Answer: Situations 2,3, and 4 require empirical methods.

3. Theoretical Probability

In the K-6 *Syllabus*, terms such as outcome and event are used but not defined. Sample space is defined, but no mention is made of outcome set. A set theory approach to probability is not shown.

An *outcome* is a result of some activity or experiment. In the tossing of a coin, "heads" is one outcome, and "tails" is another outcome. In the rolling of a die, there are six outcomes, namely 1,2,3,4,5, and 6.

a. Sample space or outcome set

A sample space (also called an outcome set) is a listing of all possible outcomes of an experiment. An *event* describes a situation related to the experiment. For example, in rolling a die,

Sample space, or outcome set	:	1,2,3,4,5,6
Event A, rolling a "3"	:	3
Event B, rolling an "even number"	:	2,4,6
Event C, rolling a "number less than 3"	:	1,2

Through the use of the theoretical definition of probability, $P(E) = {}^{f}/_{n}$, it is seen that the *probability of an event* is the ratio of the number of favorable outcomes to the total number of possible outcomes. Therefore:

$P(3) = {}^{1}/_{6}$ *or* $P(A) = {}^{1}/_{6}$

P(even number) $= {}^{3}/_{6}$ *or* $P(B) = {}^{3}/_{6}$

P(number less than 3) $= {}^{2}/_{6}$
or $P(C) = {}^{2}/_{6}$

Alternate Approach:

b. Event

From a set theory approach, the sample space (or outcome set) is the set of all possible outcomes, represented by S. An event is any subset of the sample space, represented by E. The definition of probability is the same. However, since the number of elements in the various sets is represented by n(E) and n(S), the rule may be written as $P(E) = n(E)/n(S)$.

Regardless of the approach taken, only one rule should be stated for the theoretical probability of an event. Concepts rather than symbolism should be the primary concern. Fractions need not be expressed in simplest form.

Example: A spinner contains 5 regions, numbered 1 through 5. The arrow has an equally likely chance of landing on any region. If the arrow is spun, find the probability that it lands on an odd number.

Sample Space: 1,2,3,4,5 or Sample Space S = {1,2,3,4,5}

Event A: 1,3,5 or Event A = {1,3,5}

$P(A) = {}^{f}/_{n} = {}^{3}/_{5}$ $P(A) = {}^{n(A)}/_{n(S)} = {}^{3}/_{5}$

Answer: $^{3}/_{5}$

c. Random selection

A random selection is one made without looking, or one with no special selection involved. The students should solve both standard and nonstandard problems to determine simple probabilities based upon random selections with fair objects. Standard problems include selecting a colored marble from an urn, flipping a coin, rolling a die, spinning an arrow, picking a name from a hat, etc.

Example: A standard deck of 52 cards is shuffled, and one card is drawn. Find the probability that the card is: (a) an ace (b) red (c) the jack of diamonds.

Answer: (a) $^{4}/_{52}$ (b) $^{26}/_{52}$ (c) $^{1}/_{52}$

Example: A class contains 16 girls and 12 boys. The teacher calls on students at random. Find the probability that the first person called upon is: (a) a girl (b) a boy

Answer: (a) $^{16}/_{28}$ (b) $^{12}/_{28}$

Example: Jack guesses the answer to a multiple choice question in which 4 choices are given. What is the probability that he guessed *in*correctly?

Answer: $^{3}/_{4}$

4. Range of Probabilities

In the K-6 *Syllabus*, events that are certain and events that are impossible are defined. Events that occur "sometimes" are described as having chances that could be expressed as proper fractions.

a. Certainty

If all possible outcomes are favorable, then the probability of the favorable outcomes occurring is $^{1}/_{1}$ or 1.

From a set theory approach, event E must contain every element of the sample space S. Therefore, $n(E) = n(S)$, and $P(E) = n(E)/n(S) = n(S)/n(S) = 1$.

Example: A fair die is tossed. What is the probability of rolling a number less than 7?

Answer: 1

Example: What is the probability that a multiple of 6 is also a multiple of 3?

Answer: 1

b. Impossibility

If none of the outcomes is favorable, then the probability of the favorable outcomes occurring is $^{0}/_{1}$ or 0.

From a set theory approach, event E is the empty set. Therefore, $n(E) = n(\{\ \}) = 0$, and $P(E) = {}^{n(E)}/_{n(S)} = {}^{0}/_{n(S)} = 0$.

Example: An urn contains 5 marbles: 3 red and 2 blue. What is the probability of selecting a yellow marble from this urn?

Answer: 0

Example: What is the probability that a multiple of 2 is an odd number?

Answer: 0

c. Probabilities between zero and one

Events which are neither impossible nor certain have a probability between zero and one. In these cases, $f < n$, or the number of favorable outcomes, is less than the total number of possible outcomes. Since both f and n are greater than zero, $P(E) = {}^{f}/_{n}$ is a rational number between zero and one.

From a set theory approach, $E \neq \{\ \}$, and $E \neq S$, but E is a subset of S. Therefore, $0 < n(E) < n(S)$, and $P(E) = {}^{n(E)}/_{n(S)}$ is a rational number between zero and one.

Regardless of the approach taken, the following generalization should be understood by all students. *The probability of any event is a value from zero to one.* The symbolism for this statement, however, is not required:

$$0 \leq P(E) \leq 1$$

d. Sum of the probabilities

The sum of the probabilities of the *simple events* (or *singleton events*) of the entire sample space is 1.

For example, when a coin is tossed, $P(H) = {}^{1}/_{2}$ and $P(T) = {}^{1}/_{2}$. Therefore, $P(H) + P(T) = {}^{1}/_{2} + {}^{1}/_{2} = 1$. Also, when a fair die is tossed, it is true that $P(1) + P(2) + P(3) + P(4) + P(5) + P(6) = {}^{1}/_{6} + {}^{1}/_{6} + {}^{1}/_{6} + {}^{1}/_{6} + {}^{1}/_{6} + {}^{1}/_{6} = 1$.

e. Probability of complement

The probability of a favorable event occurring added to the probability of its *not occurring* is 1.

From a set theory approach, elements not found in E are in the *complement* of E. Therefore, $P(E) + P(\text{not } E) = 1$, or $P(E) + P(\bar{E}) = 1$.

For example, the spinner is divided into 4 regions, numbered 2,3,5, and 7. Each region is equally likely to occur when the arrow is spun. Therefore:

$P(A) = P(\text{even number}) = {}^{1}/_{4}$

$P(\text{not } A) = P(\text{not even number}) = {}^{3}/_{4}$

$P(A) + P(\text{not } A) = {}^{1}/_{4} + {}^{3}/_{4} = 1$

Example: Today's weather forecast calls for a 40% chance of rain. (a) What is the probability, expressed as a

fraction, that it will rain today? (b) What is the probability that it will not rain today?

Answer: (a) P(rain) = 40% = $^{40}/_{100}$, or $^{2}/_{5}$

(b) P(no rain) = 1 – P(rain) = 1 – $^{40}/_{100}$ = $^{60}/_{100}$, or $^{3}/_{5}$

The students should solve problems including various concepts from this section.

Example: Mrs. Chang has 3 quarters, 1 dime, and 2 nickels in her purse. She takes a coin at random. Find the probability that the coin is worth: (a) exactly 25 cents (b) exactly 5 cents (c) exactly 50 cents (d) more than 5 cents (e) less than 30 cents (f) any value except 10 cents.

Answer:
(a) $^{3}/_{6}$ (b) $^{2}/_{6}$ (c) 0 (d) $^{4}/_{6}$ (e) 1 (f) $^{5}/_{6}$

5. Probability of Two Independent Events

In the K-6 *Syllabus*, independent events are defined, and sample spaces are displayed by tree diagrams and listings. The multiplication principle is presented. Here we will refer to it as the counting principle.

In grade 7, two methods should be presented to study the probability of independent events:

- A procedure whereby the sample space is carefully constructed, and the theoretical rule of probability is applied to this sample space;
- A procedure wherein the probability is defined to be a product of the independent probabilities.

In grade 7, applications will be limited to *two* independent events. Although the counting principle and the product rule for independent probabilities can be extended to three or more independent events, problems of this nature will be delayed until grade 8.

a. Definition of independent events

Independent events are those in which one event has no influence on another.

For example, getting a head on the flip of a coin and a 4 on the roll of a die are independent events. Note that getting a head on one flip of a coin and getting a head on a second flip of the same coin are independent events.

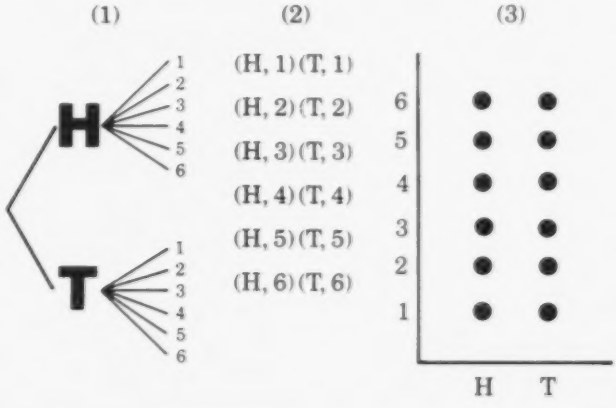
b. The counting principle (multiplication principle)

The counting principle states that if one task can be done in M ways and another in N ways, there are M × N ways to do the two tasks, usually in the order given. (In grades K-6, this principle was referred to as the multiplication principle, but counting principle is the more commonly used term. Mathematicians call this rule the counting principle because it offers shortcuts to elaborate counting procedures.)

For example, a coin can fall in 2 ways, and a die in 6 ways. By the counting principle, the coin and die can land together in 2 × 6 ways, or 12 ways.

c. Sample spaces for two independent events

By the counting principle, the sample space for tossing a coin and a die simultaneously contains 12 outcomes, which may be *displayed* in various ways:



(1) Tree diagram (2) List of ordered pairs (3) Graph of ordered pairs

Once the sample space for two independent events has been constructed in any manner shown above, the theoretical rule for probability can be applied to situations involving this sample space.

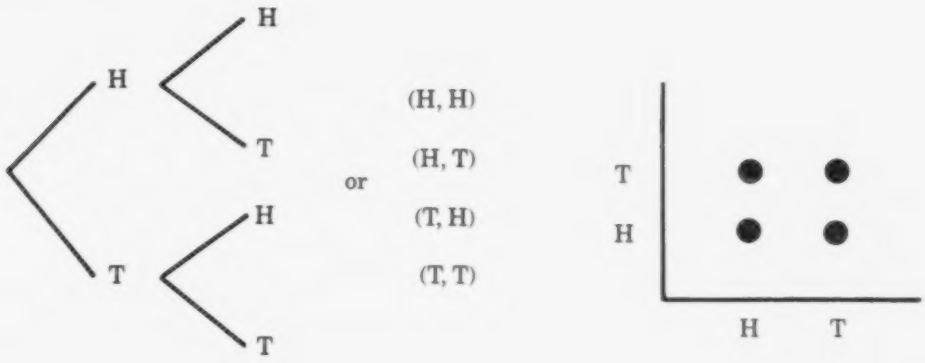
Example: What is the probability that when a coin is flipped and a die is tossed the result will be a head and a 4?

Of the 12 outcomes in the sample space, only 1 outcome indicates both a head and a 4. By the rule for probability:

P(head and 4) = $^{1}/_{12}$

Answer: $^{1}/_{12}$

Example: If two coins are tossed, what is the probability that both land heads up? Construct the sample space; note any event with two heads.

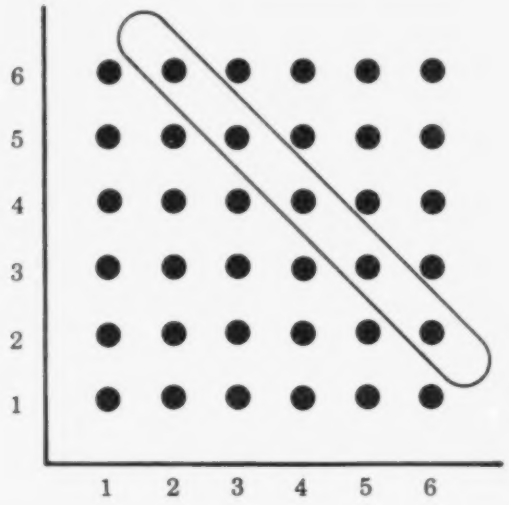


Apply the rule for probability: P(two heads) = 1/4

Answer: 1/4

Example: If two fair dice, each numbered 1 through 6, are tossed, what is the probability of rolling a sum of 8?

The sample space contains 36 outcomes, as shown in the graph. Of these, the 5 enclosed outcomes have a sum of 8.



Answer: P(sum is 8) = 5/36

Example: A spinner is divided into 4 regions, numbered 1 through 4. When spun, the arrow is equally likely to fall into any of the regions. If the arrow is spun twice, what is the probability that the first number is less than the second?

Construct the sample space; note pairs in the event:

(1,1)	(2,1)	(3,1)	(4,1)
(1,2)	(2,2)	(3,2)	(4,2)
(1,3)	(2,3)	(3,3)	(4,3)
(1,4)	(2,4)	(3,4)	(4,4)

Answer: P(first < second) = 6/16

Many other examples can be found in standard textbooks.

d. Product of independent probabilities

Introduce a second approach to similar problems in probability: For independent events, the probability of both occurring is the product of their independent probabilities. (This is equivalent to the counting principle, applied to probabilities.) Redo some problems presented earlier, before introducing more complicated exercises.

Example: What is the probability that when a coin is flipped and a die is tossed, the result will be a head and a 4?

P(head) = 1/2 P(4 on die) = 1/6

Therefore, P(head and 4) = P(head) × P(4)

= 1/2 × 1/6

= 1/12

Answer: 1/12

Example: If 2 coins are tossed, what is the probability that both land heads up?

P(2 heads) = P(head) × P(head)
= $1/2 \times 1/2 = 1/4$

Answer: $1/4$

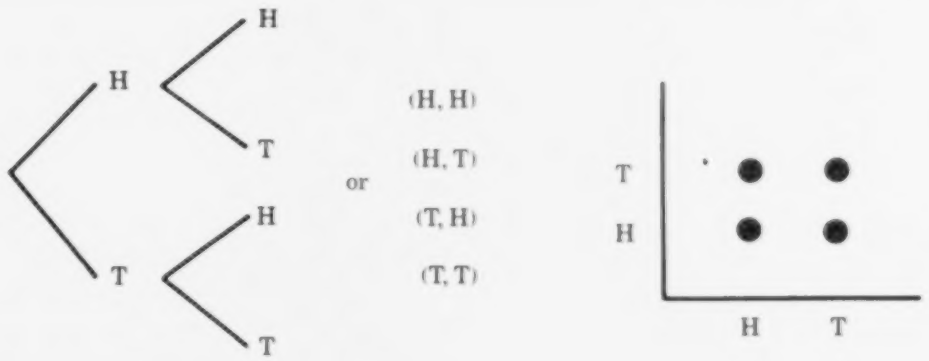
Example: Alberto visits an ice cream parlor which sells 31 flavors of ice cream, including vanilla. They have caramel, butterscotch, marshmallow, and hot fudge toppings. Alberto is offered a free sundae that was made by mistake. What is the probability that the sundae is his favorite, vanilla ice cream with hot fudge? Assume random selections.

P(vanilla with hot fudge) = P(vanilla) × P(hot fudge)
$= 1/31 \times 1/4 = 1/124$

Answer: $1/124$

The product of independent probabilities can pose a problem when various orders are involved in an event. In such cases, the probability is found for each order, and then the sum of these probabilities is taken. Consider two approaches to the following problem.

Example: If two coins are tossed, what is the probability that a head and a tail appear? (No order is implied.)



P (H, T in that order) = P (H) × P (T) = $1/2 \times 1/2 = 1/4$
P (T, H in that order) = P (T) × P (H) = $1/2 \times 1/2 = 1/4$

P (H and T in any order) = $1/4 + 1/4 = 2/4$

Answer: $2/4$

Example: Cavallaro has a batting average of .300, and Bixhorn is a .250 hitter. Cavallaro and Bixhorn bat in that order. Find the probability that: (a) both players get hits; (b) neither player gets a hit; (c) at least one player gets a hit.

(a) P(both hit) = P(Cavallaro hits) × P(Bixhorn hits)
$3/10 \times 1/4 = 3/40$

(b) P(neither hits) = P(Cav. no hit) × P(Bix. no hit)
$7/10 \times 3/4 = 21/40$

(c) "At least one hit" means "anything except no hits." Use the complement.
P(at least one hit) = 1 – P(neither hits)
$1 - 21/40 = 19/40$

B. STATISTICS

In grades K-6, the students learned to collect and organize data, determine the mean (average) and range, and construct various graphs, namely the bar, line, picture, circle, and double bar.

1. Definition

Statistics is the study of numerical *data*, that is, pieces of numerical information.

a. Steps in a statistical study

The typical steps in a statistical study are:

(1) the *collection* of data
(2) the *organization* of these data into tables and graphs

(3) the *analysis* of the data to present the findings of the study.

In some cases, a fourth step is added when the analyzed data are used to establish patterns, to predict trends and future events, and to help in the decision-making process.

b. Sample

Data are collected in various ways, including questionnaires, interviews, logs, and diaries. In many cases, only a sample of the data is needed to study a large population. For example, a manufacturer tests only a sample of the light bulbs to determine the life span of the product sold. Every sample should be unbiased, and should be taken at random. Also, to find out which sport is the most popular in the school, it is not necessary to question every student. However, the sample should not consist of students at a baseball game. A better sample might consist of four students from each homeroom, perhaps the first four in alphabetical order to obtain a good random selection.

2. Organization of Data

Statistics is often best studied through class projects, individual student projects, and surveys that are meaningful to the students. The example used in this guide is merely an illustration.

Data are usually collected in an unorganized and random manner. Consider these 25 test scores as an example:

73, 68, 83, 89, 80, 100, 92, 45, 80, 90, 76, 78, 80, 100, 65, 95, 84, 81, 77, 67, 99, 100, 79, 81, 94.

a. Range

The range is the difference between the largest and smallest scores in the set of data. In this case, the range is 100 – 45 = 55.

b. Intervals of equal length

The data are organized or *grouped* into a table by using intervals of equal size or equal length. Every score must fall into one and only one interval, so the intervals must not overlap. An interval of 91-100 contains 10 scores, as does the interval 81-90. Each interval is of length 10, found by subtracting the starting point of one interval from the starting point of the next higher interval.

The number of intervals should be between 5 and 15 for the most effective grouping of data and, eventually, for the presentation of a graph. A score is noted by placing a *tally* (|) in the appropriate interval; groups of five tallies (𝍸) make counting easier.

INTERVAL	TALLIES
91–100	𝍸 ||
81–90	𝍸 |
71–80	𝍸 |||
61–70	|||
51–60	
41–50	|

c. Frequency distribution

The tallies are converted to counting numbers which tell us the *frequency*, or the number of scores, in each interval. The frequency distribution shows us how the scores fall into the various intervals in the study. This table is an example of *grouped data*.

INTERVAL	FREQUENCY
91–100	7
81–90	6
71–80	8
61–70	3
51–60	0
41–50	1

The students may determine the intervals to be used, or the teacher may assign intervals for data. In either case, the students should have practice in grouping data to find the frequency distribution. If a student selects intervals of 70-79, 80-89, and so forth, the student must understand why 90-100 is not an acceptable interval. Similarly, the student should understand that different

interval groupings will produce different frequency distributions.

As an example, the same set of ungrouped data representing 25 test scores is distributed into a new set of intervals. A completely different frequency distribution emerges.

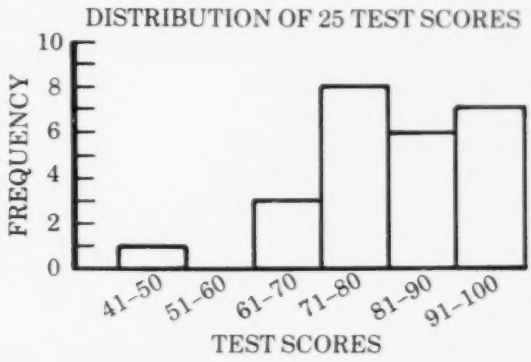
INTERVAL	TALLIES	FREQUENCY
100–109	𝍸	3
90–99	𝍸	5
80–89	𝍸 |	8
70–79	𝍸	5
60–69	|	3
50–59		0
40–49	|	1

3. Histogram

Data that have been organized into a frequency distribution can be displayed graphically. A *histogram* is a bar graph in which the bars are placed next to each other to show that as one interval ends, the next interval begins. A histogram is drawn to show "changes in the same item," such as the distribution of test scores.

Consider the first frequency table for the distribution of test scores. A histogram of these data is drawn, to include: (1) a vertical scale indicating the frequencies; (2) a horizontal scale showing intervals of equal length; and (3) a title for the graph.

INTERVAL	FREQUENCY
91–100	7
81–90	6
71–80	8
61–70	3
51–60	0
41–50	1



The students should not only construct histograms to present the findings of statistical studies; they should also answer questions based upon graphic displays.

Example: Answer the following questions based upon the histogram shown:

(a) In which interval is the greatest frequency found?
(b) How many students received test scores between 61 and 70?
(c) How many students scored higher than 80?
(d) What percent of students received scores higher than 80?

Answer: (a) 71-80 (b) 3 (c) 13 (d) 52%

4. Measures of Central Tendency

In K-6, the students used the word "mean" to indicate an average. They have not, however, studied the median or the mode, nor have they used the term "measure of central tendency." In grade 7, the three measures of central tendency noted here are to be studied using only small sets of data. These measures will be applied to grouped data in grade 8.

Measures of central tendency (the mean, the median, and the mode) are numbers that "tend" to fall somewhere in the "center" of a set of organized data.

a. Mean

The mean is a numerical average. To find the mean of a set of N numbers, add the numbers and divide the sum by N.

Example: The weekly salaries of 7 people working for ABC Toys, Inc. are: $160, $180, $160, $230, $210, $160,

and $1700. Find the mean salary of these 7 people.

$$\text{Mean} = \frac{160 + 180 + 160 + 230 + 210 + 160 + 1700}{7} = \frac{2800}{7} = 400$$

Answer: Mean = $400.

Discuss the problem above. Although the average or mean salary is $400, 6 of the 7 employees fall below this average. The one high salary distorts the "average" salary so that people may believe that the workers are earning much more than they really are. In this example, the mean does not truly reflect the salaries being earned.

Let us consider other measures of central tendency.

b. Median

The median is the "middle score" for a set of data. It is taken only when the data are arranged in numerical order (either from highest to lowest, or from lowest to highest).

Example: Use the data reported earlier to find the median weekly salary of the 7 people employed at ABC Toys, Inc.

The salaries are arranged in numerical order. The median is $180, the score in the middle of this distribution.

$1700
$ 230
$ 210
$ 180 Middle
$ 160
$ 160
$ 160

Answer: Median = $180.

Ask the students why for this set of data this measure is a better indication of the salaries earned. (Excluding the employee earning a median salary of $180., half of the remaining workers have salaries above this amount and half have salaries below this amount.)

If there are two middle numbers, the median is the mean (average) of these two numbers.

Example: What is the median height for the following data?

145 cm, 150 cm, 150 cm, 160 cm, 165 cm, 170 cm

The two middle scores are 150 and 160. The mean of these two numbers is $^{150+160}/_{2} = {}^{310}/_{2} = 155$

Answer: Median = 155 cm

Note: In such cases the median may not always be one of the original scores.

c. Mode

The mode is the score that appears most often in a set of data. Some stipulations concerning the mode are stated after the following example.

Example: Weekly salaries of 7 employees at ABC Toys, Inc. are: $160, $160, $160, $180, $210, $230, $1700. Find the mode for this set of data.

Answer: Mode = $160.

Ask the students if in this example the mode is a better indication of the salaries being earned than the mean average of $400. Since most workers earn $160, the mode does reflect this set of data with some meaning. Note that it is helpful to arrange the data in numerical order before finding the mode.

Some sets of data have more than one mode; others have no mode. Consider these stipulations to find the mode:

(1) If one score appears most often, that score is the mode.
(2) If two or more scores appear the same number of times, and more than any other score, then each of these scores is a mode.
(3) If no number occurs more than once, or if every score appears an equal number of times, there is no mode.

Consider the following examples.

Example: Find the mode for the following set of ages in a class:

11,11,11,12,12,12,12,12,12,13,13,13,13,13,13,14

Answer: There are two modes, namely 12 and 13.

Note that both 12 and 13 are reported as modes; it is not correct to write the mean (average) of these modes.

Example: Find the mode for the following data which indicate the number of people in different apartments in a building:

1,1,1,1,2,2,2,2,3,3,4,4,4,4,5,6,8,8

Answer: 1,2, and 4 (three modes)

Example: Find the mode for the following data which indicate the number of words in student essays:

35,78,84,91,95,102,115,120,121,125,130,131,286

Answer: No mode

Example: Find the mode for the data:
8,8,10,10,14,14,20,20.

Answer: No mode, since every score appears an equal number of times.

d. Problem solving

Variations of standard problems involving all three measures of central tendency noted here should be solved by the students. Verbal problems should be incorporated with these concepts as much as possible.

The students should also be able to examine several sets of data and to determine which measure(s) of central tendency (mean, median, or mode) best represents the sets of data.

Discuss the fact that the three measures of central tendency may be close in value for some sets of data, and may be far apart and very different for other sets of data. The meaning of these situations is studied in future grades when statistics is more deeply analyzed.

Example: Find the mean, median, and mode for Debbie's scores on her last 6 science tests: 30,76,85,91,93,93.

Answer: Mean = 78, Median = 88, Mode = 93

Example: Farmer Brown delivered 10 dozen eggs in cartons to a local market. The number of uncracked eggs per carton is: 12,12,12,12,12,12,12,12,11,11. For these data, find the mean, median, and mode.

Answer: Mean = 11.8, Median = 12, Mode = 12

Example: Find the mean, median, and mode for the number of pages in notebooks that Jay purchased: 80,80,128,196,196

Answer: Mean = 136, Median = 128, Mode = 80 and 196

Example: Rosa received grades of 83, 87, and 94 on her last three tests in social studies. What must she earn on her next test so that the mean of her four grades is 90?

Answer: 96

Example: For which set of data are the mean, median, and mode all equal?

(1) 1,1,1,3,4 (2) 1,2,4,4,4
(3) 1,4,5,5,6,9 (4) 3,3,12

Answer: (3)

Example: What is the median age in a classroom where 16 students are 12 years old and 13 students are 13 years old?

Answer: 12

Example: The set of data 3,7,8,N,7,8 is given. In each part, find all possible values of N so that: (a) there is no mode; (b) there is only one mode; (c) there are two modes.

Answer: (a) 3 (b) 7 or 8 (c) any number except 3,7, or 8

V. ALGEBRA AND COORDINATE GEOMETRY

The following introduction to algebra emphasizes algebra as a language. This approach stresses the use and interpretation of algebraic symbols. In this way, the students will have a firm foundation upon which to build the learning of the more formal side of algebra.

In grade 6, the students wrote and solved open sentences which were first-degree equations in one variable. Letters and frames were used as placeholders. Inverse operations were mentioned, but terminology was kept to a minimum level. In grade 7, those algebraic skills that help the students to develop an orderly and systematic approach to problem solving will be expanded. The material is organized here into three general topics: algebraic expressions and sentences; first-degree equations and inequalities in one variable; and functions (first-degree equations with two variables) and the coordinate plane.

The students should begin to write and solve open sentences while working with word problems. Since algebra is the language of science, applications to science will be employed wherever possible – providing they are understandable to the students and are not too complex or too trivial. The work should be of some consequence; the students should begin to appreciate the fact that algebra is important because of what it enables people to do.

A. ALGEBRAIC EXPRESSIONS AND SENTENCES

1. Concepts Through Applications

Concepts and terminology should be developed as the students begin to analyze the outcomes of given situations, experiments, and word problems.

Example: With each step that Barbara takes, she covers a distance of 30 centimeters. Find the distance covered in centimeters by 1, 2, 3, and 4 of Barbara's steps.

Answer: With one step, 30•1 or 30 cm
With two steps, 30•2 or 60 cm
With three steps, 30•3 or 90 cm
With four steps, 30•4 or 120 cm

a. Variables, constants, and domains

In analyzing this problem, the students should see that each expression is of the form 30•N, where N is a variable or placeholder, and 30 is a constant. The variable N represents a member of the set {1, 2, 3, 4}, called the domain or replacement set.

b. Open sentences vs. algebraic expressions

Distinctions should be made between sentences (which contain complete thoughts) and expressions or phrases (which are parts of sentences). A sentence containing one or more variables is an open sentence; when the variables are replaced by elements from the domain, the sentences may be judged to be true or false. An expression containing one or more variables is an open expression or an algebraic expression; when its variables are replaced, the result is a numerical value or a numerical expression.

Open sentence: 30•N = 90 Algebraic expression: 30•N
True sentence: 30•3 = 90 Numerical expression: 30•3
False sentence: 30•4 = 90

A domain may be extended or restricted to fit a given problem. The domain of whole numbers, or the domain of positive rational numbers, can be used in the next two examples.

Example: Barbara takes exactly 90 steps to go from her English classroom to her math classroom. What is the distance between these rooms?

(Evaluate the expression 30•N when N = 90)

Answer: 2700 cm, or 27 m

Example: Barbara covers the length of a classroom in 30 steps, and the width in 25 steps. What are the dimensions of the classroom?

(Evaluate the expression 30•N for N = 30, and N = 25)

Answer: 900 cm by 750 cm, or 9 m by 7.5 m

2. Translating Verbal Phrases into Algebraic Form

Verbal problems and situations involving variables should be presented to the students. It is helpful for the

students to think of similar but related problems involving numbers and, once the arithmetic process is recognized, apply the same method to write algebraic expressions.

a. Forming algebraic expressions

As a first step, the students might translate verbal phrases involving addition, subtraction, multiplication, and division into algebraic form. For example:

Verbal Phrase	Algebraic Expression
(1) 5 more than x	$x + 5$
(2) a number decreased by 4	$N - 4$
(3) y subtracted from 20	$20 - y$
(4) the product of 8 and k	$8 \times k$, or $8k$
(5) the quotient of c and d	$c \div d$, or c/d
(6) 3 times m, increased by 2	$3m + 2$
(7) 3 times the sum of m and 2	$3(m + 2)$

The students should realize that a simple algebraic expression may represent several different verbal phrases. As an example, $x + y$ means:

x plus y y is added to x y more than x

the sum of x and y x increased by y

Eventually, algebraic expressions should be formed based upon verbal problems. Once an algebraic expression is written, questions may be posed leading to an evaluation of the expression or a reinforcement of concepts.

Example: Mike is 3 years older than Fred. If Fred is x years old, represent Mike's age in years.

Answer: $x + 3$

Example: Alice is 4 cm shorter than Roberta. What is Roberta's height in centimeters if Alice is h centimeters tall?

Answer: $h + 4$

(How tall is Roberta if Alice is 160 cm tall?)

Example: Len had a dozen eggs and used n of them to make an omelet. How many eggs are left?

Answer: $12 - n$

(What could be the domain of n?)

Example: If a notebook costs 79 cents, what is the cost in cents of k notebooks?

Answer: $79k$

Example: Represent the number of quarts in p pints.

Answer: $p/2$, or $p \div 2$

(How many quarts are there in 4 pints? in 5 pints? in 1 pint?)

Example: Represent the number of hours in A days and B hours.

Answer: $24A + B$

Example: A shirt costs 14 dollars, and a pair of jeans costs 20 dollars. What is the cost in dollars of x shirts and y pairs of jeans?

Answer: $14x + 20y$

Example: A 2-liter bottle contains 2000 mL of soda. A glass holds 200 mL of soda. Represent the number of milliliters of soda left in the bottle after filling k glasses of soda from the bottle.

Answer: $2000 - 200k$

(What is left in the bottle after filling 3 glasses? What could be the domain of k in this problem?)

b. Evaluating algebraic expressions

The order of operations may be reviewed through direct substitutions of numbers for variables in algebraic expressions. On a more challenging level, questions could be posed concerning evaluations using given domains.

Example: Evaluate $8x+2x$ when $x = 17$.

Answer: 170

Example: If $x = 5$ and $y = 2$, which expression has the smallest value?

(1) xy (2) $2x - y$ (3) $2x/y$ (4) $x - y^2$

Answer: (4)

Example: For what members of the domain {1, 2, 3, 4, 5} is the expression $\frac{n+1}{2}$ equal to a whole number?

Answer: 1, 3 and 5

B. FIRST DEGREE EQUATIONS AND INEQUALITIES IN ONE UNKNOWN

1. Forming Open Sentences

Simple applications should be used to help the students write, solve, and check first degree equations and

inequalities in one unknown. The terminology should be an outcome of the analysis of these word problems.

Example: After Gloria deposited $26 in her bank account, she had a total savings of $408. What amount did she have in the account before the deposit?

Let x = amount before deposit.

$x + 26 = 408$

(Methods of solution are shown in the section that follows.)

Answer: $382

In analyzing an algebraic approach to the problem, the students should realize that an equation was written in which the left-hand side or left-hand member is equal to that of the right-hand. An *equation* is a sentence that uses an equal sign (=) to state that two expressions are equal. Any variable used in the equation should be identified before the equation is written.

2. Solving Open Sentences that are Equations

Equations, such as the one just presented, may be solved by either of two methods: *replacement* or *the use of inverse operations.*

a. Replacement

A *solution* is any element of the domain that replaces the variable in an open sentence to make the sentence true. The *solution set* is the set of all solutions of the equation or the inequality. If no domain is stated, it is assumed to be all numbers known by the students.

Write the equation: $x + 26 = 408$

Replace x with 382: $382 + 26 = 408$

$408 = 408$ True

All other replacements yield false sentences.

Write the solution: x = 382, or solution set = {382}

b. Inverse operations

Inverse operations may be used to transform an equation to a simpler equivalent equation where the solution is more obvious. Addition and subtraction are inverse operations. Multiplication and division are inverse operations for nonzero numbers. Ask the students how the following approach compares to one in which two numbers are subtracted to find the answer.

$$\begin{array}{rcr} x + 26 = 408 & \text{or} & x + 26 = 408 \\ x + 26 - 26 = 408 - 26 & & -26 \quad -26 \\ x = 382 & & x = 382 \end{array}$$

When this approach is used, it is recommended that a *check* be shown involving a simple replacement or substitution.

Check: $x + 26 = 408$

$382 + 26 = 408$

$408 = 408$

Example: Helen and Arthur have coin collections. Helen has 3 times as many coins as Arthur has. If Helen has 255 coins, how many coins does Arthur have in his collection?

Let x = number of coins Arthur has

Let 3x = number of coins Helen has

$\frac{1}{3}\,3x = \frac{1}{3}\,255$ or $\frac{3x}{3} = \frac{255}{3}$

$x = 85$ $\qquad$ $x = 85$

Check: $3x = 255$

$3(85) = 255$

$255 = 255$

Answer: 85, or solution set = {85}

The students should recognize that the domain in this problem may be either the set of counting numbers or the set of whole numbers.

The examples which follow illustrate other applications in which first degree equations are formed, solved, and checked. Percent problems, first seen in III-C3 and 4, are reexamined from an algebraic viewpoint.

Example: After Mr. Weinrich lost 7 kg, he weighed 68 kg. What was his original weight?

Answer: 75 kg (Use $w - 7 = 68$)

Example: The diameter of the moon is approximately $\frac{1}{4}$ the diameter of the earth. If the moon's diameter is 3180 km, find the approximate diameter of the earth.

Answer: 12,720 km (Use $\frac{1}{4}\,d = 3180$)

Example: There are 18 girls in a class. If $\frac{2}{3}$ of the class are girls, how many students are in the class?

Answer: 27 (Use $\frac{2}{3}\,c = 18$)

Example: If Mr. Taslitz drives at an average rate of 40 miles per hour, how long will it take him to travel 100 miles?

Answer: $2\frac{1}{2}$ hours (Use RT = D, specifically 40T = 100)

Example: The rate of sales tax is 8%. If Ms. Schmeling buys a $420 TV set, what must she pay in sales tax?

Answer: \$33.60 (Use $^{8}/_{100}$ x 420 = t, or 420 (.08) = t)

Example: Mrs. Henez earns 5% commission on every car she sells. Find the selling price of a car that earned her a commission of \$490.

Answer: \$9,800 (Use .05s = 490)

Example: Connie bought a shirt on sale. The clerk deducted \$3 from the original price of \$12. Find the percent of discount.

Answer: 25% (Use 12 • $^{x}/_{100}$ = 3)

Example: A piece of cloth 98 inches wide is cut into 4 equal parts, with a single 6-inch wide piece remaining. Find the width of one of the equal parts.

Answer: 23 inches (Use 4 w + 6 = 98)

3. Solving Open Sentences that are Inequalities

An *inequality* is a sentence that uses a symbol (usually $<$, $>$, $\leq$, or $\geq$) to state that two expressions are not equal. Replacement and inverse operations are used in solving inequalities.

By replacement:

Domain = Whole Numbers

Solve $x+4<7$

$0+4<7$	True
$1+4<7$	True
$2+4<7$	True
$3+4<7$	False
$4+4<7$	False

Solution, 0, 1, 2

or, solution set = {0, 1, 2}

or,

Simplifying by inverse operations:

Domain = Whole Numbers

Solve $x+4<7$

$x+4-4<7-4$

$x<3$

The condition has been transformed to a simpler equivalent inequality.

Solution: 0, 1, 2

or, solution set = {0, 1, 2}

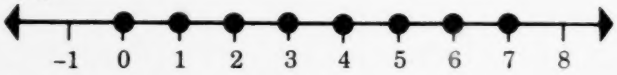
or,

Simple inequalities, including those resulting from word problems, should be presented and solved.

Example: If the number of computers at school is doubled, there would still be less than 15. How many computers are now in the school?

Answer: 0,1,2,3,4,5,6,7 (Use $2c < 15$)

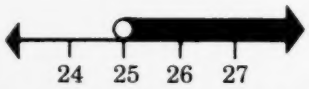
or,



Example: In the last hour, the wind decreased 10 mph. It is still blowing more than 15 mph. Find the rate of the wind one hour ago.

Answer: The wind was greater than 25 mph. (Use $w-10 > 15$)

or,

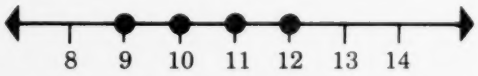


Some word problems, although they appear to be relatively easy to solve at first glance, require a great deal of thought. In each of the following examples, allow time for the students to select an appropriate domain and to consider all conditions implied in the problem.

Example: After 9 people left a party, there were fewer than 4 people remaining. How many people attended the party?

Answer: 9, 10, 11, or 12 people (Use $n-9<4$)

or,



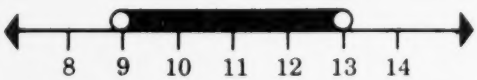
Note: The domain is the counting numbers. From $n-9<4$, it follows that $n<13$. However, since 9 people attended and left the party, it follows that 9 or more were present.

Example: After 9 inches is cut from a piece of wood, the remaining piece is less than 4 inches long. How long was the original piece of wood?

Answer: Any length greater than 9 inches and less than 13 inches.

(Use $n-9<4$)

or,



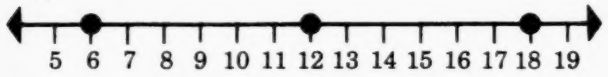
Note: The domain can consist of numbers other than counting numbers. From $n-9<4$, it follows that $n<13$.

However, since a 9-inch piece was cut away and another piece remains, the original length is more than 9 inches.

Example: A box of pencils was shared equally among 6 students so that no pencils were left in the box. If each student received fewer than 4 pencils, how many pencils were originally in the box?

Answer: 6,12,18 (Use $p/6 < 4$)

or,



Note: The domain is the counting numbers. From $p/6 < 4$, it follows that $p < 24$. However, since no pencils remain, the number in the box must be a multiple of 6.

C. FUNCTIONS AND THE COORDINATE PLANE

1. Concepts through Applications (Functions of the form y = ax)

The concept of function and all necessary terminology can be developed by analyzing simple applications. Each application in this section, whether it is a word problem or an experiment performed in the classroom, is a *function of the form* y = ax.

The students should organize and analyze the data resulting from the application, and they should prepare or present the following:

- a *table*, showing how two variables are related.
- a *sentence*, describing the function in words.
- a *formula*, describing the function in symbols.
- a *graph*, displaying a picture of the function.

The teacher should choose one or more of the following examples for a classroom presentation. One example will be developed in this *Syllabus* as a model.

Example: A student who does errands and yard work for a neighbor earns $3 per hour. Develop the function y = 3x where x = time in hours and y = pay in dollars.

Example: Cheese costs $2 per pound. Develop the function y = 2x where x = number of pounds and y = cost in dollars.

Example: A car travels at a fixed speed of 20 mph. Develop the function d = 20t where d = distance in miles and t = time in hours. (Alternate form: y = 20x; x = time in hours; y = distance in miles).

Example: The distance in miles, D, away from lightning is a function of the time in seconds, t, between the flash of lightning and the sound of thunder. Develop the formula $D = \frac{1}{5}t$. (Alternate form: $y = \frac{1}{5}x$; x = time in seconds; y = distance in miles.)

Although classroom experiments are time-consuming, they provide motivation and hands-on experience in developing the concepts in this unit.

Experiment: Weigh cream-filled cookies on a balance scale. Relate the number of cookies to their total weight. For example, if each cookie weighs 10 grams, the function y = 10x is formed where x = number of cookies and y = weight in grams.

Experiment: Find out how far a spring will stretch when uniform weights are hung from the spring. (This experiment will be of the form y = ax only over a limited range. Let x represent the number of uniform weights, and let y represent the stretch of the spring in some unit of length.)

Experiment: Using the circular bases of various objects such as waste baskets and tin cans, measure the circumference C and diameter d. Develop the formula $C = \pi d$.

Example: A student who does errands and yard work for a neighbor earns $3 per hour. As students analyze this application, they should construct the following:

a. *Table of values*

Number of Hours Worked	0	1	2	3	4
Pay in Dollars	0	3	6	9	12

b. *Sentence*

Is there a pattern to these data? Describe it in sentence form:
"The pay in dollars is three times the number of hours worked."

c. *Formula*

Change the sentence to algebraic form by using variables.
Let x = number of hours worked.
Let y = amount of pay in dollars.
The pay in dollars is three times the number of hours worked.
y = 3x

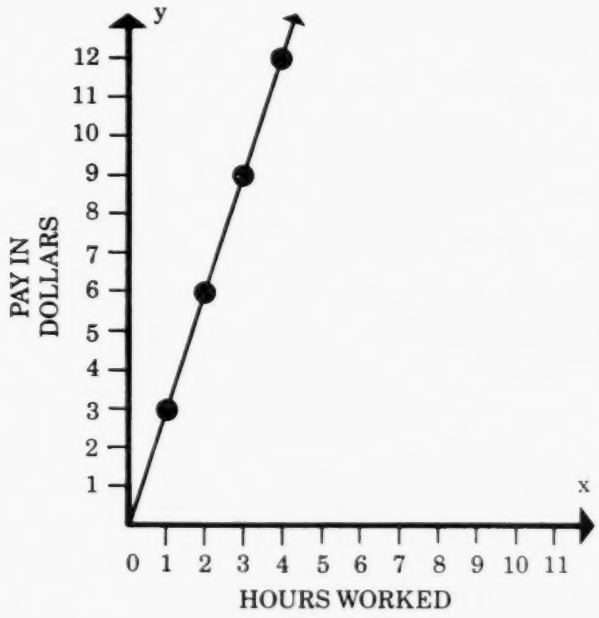
d. Coordinate graph (first quadrant)

The use of two variables, x and y, requires a graph with *two axes*. The horizontal line or axis represents x, the number of hours worked; the vertical line or axis represents y, the amount of pay in dollars. Points are plotted

using *ordered pairs* or *coordinates* of the form (x, y).

For example, (1,3) means that 1 hour of work results in a payment of 3 dollars. The axes meet at a point called the *origin* whose coordinates are (0,0).

The points determined by the ordered pairs in the table are plotted. Notice that the points can be connected by a straight line. The formula y = 3x is an example of a *linear equation* or a *linear function*.



This graph is restricted to the first quadrant because the application is meaningful only when x and y values are positive or zero. The same restriction holds in all suggested activities in this section.

e. Predictions

Prediction 1: *Given values for x, find corresponding values for y.*

To find a value for y, various procedures can be used, including substitution in the formula, reading values from the graph, and using patterns in the table.

Example: Find the pay due for 2½ hours of work.

Procedure: Write the formula,	$y = 3x$
substitute $x = 2\frac{1}{2}$, and compute.	$y = 3(2\frac{1}{2})$
Answer: \$7.50	$y = 7\frac{1}{2}$

Example: Find the pay for 6 hours of work.

Procedure: Use the graph. Locate x = 6 on the horizontal axis, then move up vertically to the line graph, and from that point move horizontally to the y-axis. When x = 6, y = 18.

Answer: \$18

Prediction 2: *Given values for y, locate corresponding values for x.*

Procedures similar to those just described can be applied to these examples.

Example: How many hours did the student work to earn \$15?

Procedure: Write the formula,	$y = 3x$
substitute y = 15, and solve	$15 = 3x$
the equation.	$\frac{1}{3}(15) = \frac{1}{3}(3x)$
	$5 = x$

Answer: 5 hours

2. Functions of the Form y = ax + b (*Optional*)

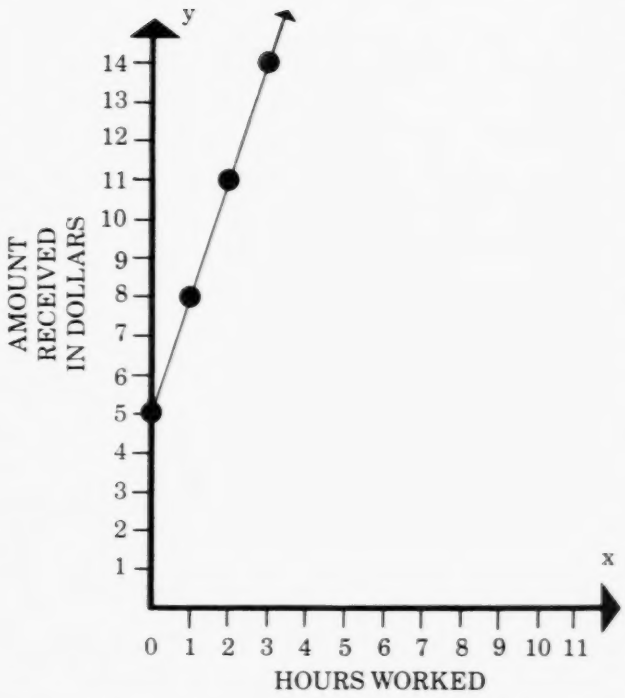
Some applications may be presented to show linear functions of the form y = ax + b. In such applications, care should be taken to restrict x and y values to positive numbers and zero.

Example: A student earns \$3 per hour for doing chores for a neighbor. This month, the student was given \$5 as a birthday gift by the neighbor. How much money did the neighbor give to the student this month?

x (Number of hours worked)	0	1	2	3
y (Amount received in dollars)	5	8	11	14

If no work is performed, the student still receives \$5 this month. Explain why the graph of the line does not pass through the origin. Write the formula:

$$y = 3x + 5$$



Discuss the meaning of each part of the formula, namely, x, 3x, 5, and y.

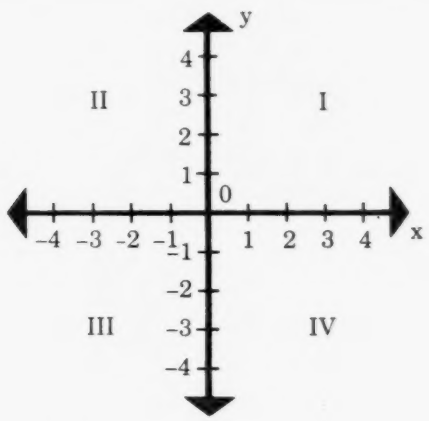
Example: Let x represent the number of cookies in a box, with each cookie weighing 10 grams. The box has a weight of 28 grams. Let y represent the total weight of the cookies and the box. Develop the formula:

$y = 10x + 28$

3. The Coordinate Plane

By extending the x-axis and the y-axis to include negative numbers, the coordinate plane can be presented. The students should plot points in all four quadrants and on both the x-axis and the y-axis. A series of such points, when connected in a specific order, may form a simple picture. In the K-6 *Syllabus*, this topic is introduced in grade 4.

Applications and word problems that result in lines which are graphed in quadrants beyond the first quadrant will not be studied in grade 7.

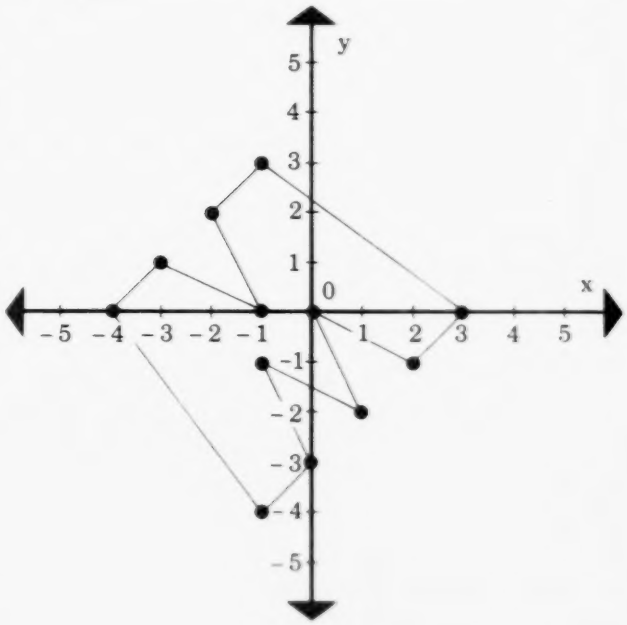


Example: Graph and label the following points on a coordinate plane.

A (0,4) B (1,3) C (0,1) D (2,2) E (3,1) F (0,-3)
G (-1,-2) H (0,0) I (-2,-1) J (-1,1) K (-3,0) L (-4,1)

Connect the points in alphabetical order and complete the picture by connecting points L and A.

Answer:



VI. COMPUTER AWARENESS

A working knowledge of computers becomes increasingly important as computer technology advances by leaps and bounds, affecting industry, the home, and the school. This strand is designed to provide the students with an introduction to this field. Basic definitions and concepts are outlined in the first section, along with mention of the computer's impact on our present-day lives.

In the second section of this unit, mathematical operations and functions are discussed from a computer viewpoint. The teacher, as well as the students, should realize that the computer is a problem-solving device, and a program is merely an algorithm or a plan for solving a specific problem. As today's students solve problems in the future, they will frequently use computers. An algorithm, however, can be studied without the use of a computer.

The students must have some knowledge of the operations and functions found in a typical programming language to enable them to construct meaningful algorithms and procedures.

The third section of this unit deals with computer programming itself. This section is optional because, in some schools, sufficient equipment may not be present. However, in schools where computers are available, the students should have some hands-on experience with these devices.

It is recognized that some schools present this material in a separate curriculum outside of the regular mathematics classroom. In such a case, it is not necessary to repeat these sections in this course of study.

Note: The teacher may present this strand early in the year so that the computer is woven throughout the entire curriculum at appropriate places. Commercially prepared programs can supplement the curriculum and increase student exposure to the computer.

A. INTRODUCTION TO COMPUTERS

This section, including time devoted to classroom discussion and/or student reports, should be completed in 2 to 3 days.

1. What Is A Computer?

A number of students have computers in their homes, and even more have operated computers and run packaged programs. Every attempt should be made to use the students' existing knowledge and experiences as a basis for developing this strand.

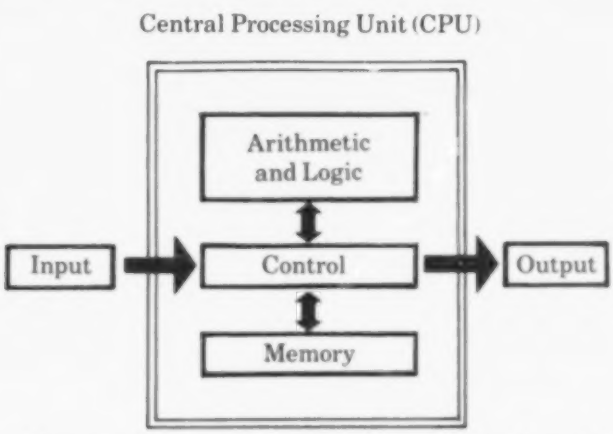
Devices such as the abacus, slide rule, and hand calculator have been used to help people compute. A *computer* is a device that can store and retrieve data and can perform computations at high speeds, without the intervention of a human being once it begins to run. To perform these computations, however, the computer must follow a detailed set of instructions, called a *program*, which is written by a person. In effect, a program is a plan for solving a problem, and the computer merely follows the plan and performs the calculations to give us the answer.

2. Components of a Modern-Day Digital Computer

The first modern-day digital computer was developed in 1945. Since that time, computer technology has experienced rapid growth. Early computers filled large rooms. Today, the entire computing capacity of the first computers built can be placed on an electronic chip that is smaller than a fingernail. Machines have become relatively compact, faster, and less expensive. These trends will continue, and it is difficult to predict the total impact that the computer will have on our lives in the future.

The *hardware* of a computer consists of the physical components that are assembled to make a computer system. The *software* is the set of programs and procedures needed to operate the computer. Hardware includes the *central processing unit* (CPU) which contains the inter-

nal components of the computer, and the *input/output* (I/O) *devices* which are used to enter and receive data.



There are five basic components in every modern-day digital computer system:

a. *Input*: the data and instructions (program) that are put into a computer system. An input device, such as a keyboard, is used to give this information or input to the computer.

b. *Control*: an internal part of the computer that carries out the instructions of a program by following a given sequence of steps and by controlling other parts of the computer.

c. *Arithmetic and logic unit*: an internal part of the computer where calculations are performed and where decisions are made based upon a comparison of data.

d. *Memory* (or *storage*): an internal part of the computer where data and programs are stored and kept. (External memory consists of information stored on other devices such as disks and tapes.)

e. *Output*: the data received from a computer, like the solution to a problem. An output device, such as a printer or a television monitor, is used to display the information or output given to us by the computer.

Ask the students how these five components of a computer differ from the processes and capabilities of the human mind when solving a problem:

- Input – We read a problem or hear it from someone; we gather facts.
- Control – Part of our brain decides which steps to follow.
- Arithmetic and Logic Unit – Another part of our brain performs computations and makes comparisons of data.
- Memory – A part of our brain stores information, or we write these facts down on paper when solving a problem.
- Output – We write the answer, or give it orally.

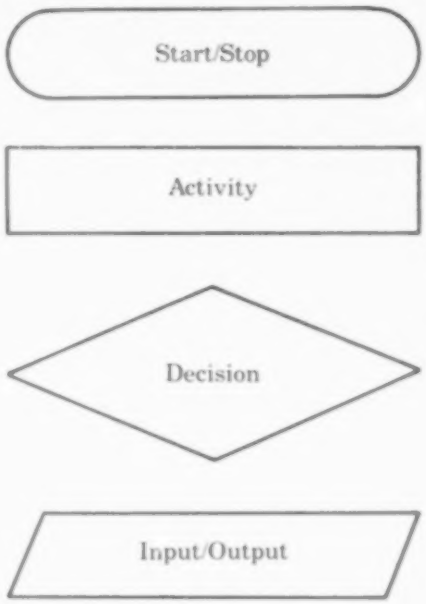
In such a comparison, the computer has the advantage of greater speed and, in some cases, greater memory, but the computer cannot "think." The computer can follow the steps to solve a problem, but it cannot create the plan needed to solve the problem. It is the "control" component of the human mind that directs the "control" of the computer by means of a carefully constructed program which tells the computer the logical steps to take.

3. Organizing a Plan for a Computer Solution

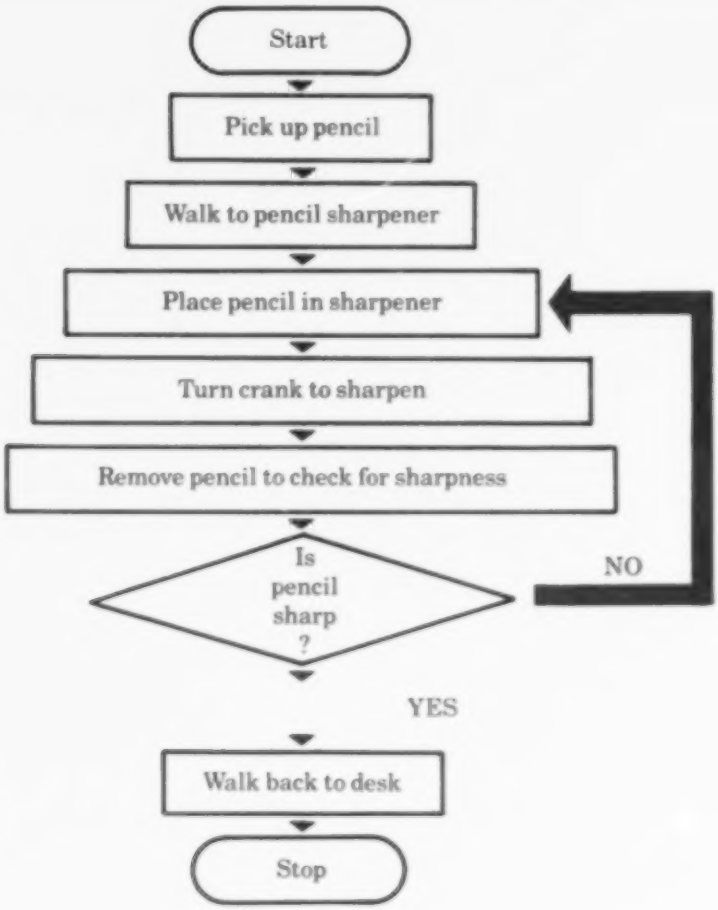
a. Flowchart

Some programmers use flowcharts, but many do not. One or two flowcharts, however, should be presented in class because these visual displays of algorithms will help the students to understand the systematic approach needed for a computer program. At least one program should be presented (see below), both as an illustration and as a means of showing how the components within a computer function.

Students should realize that a computer can perform only one operation or follow one step at a time. Instructions to a computer must be precise and orderly. A flowchart is a diagram of the logical sequence of steps needed to solve a problem. Symbols used in flowcharting include:



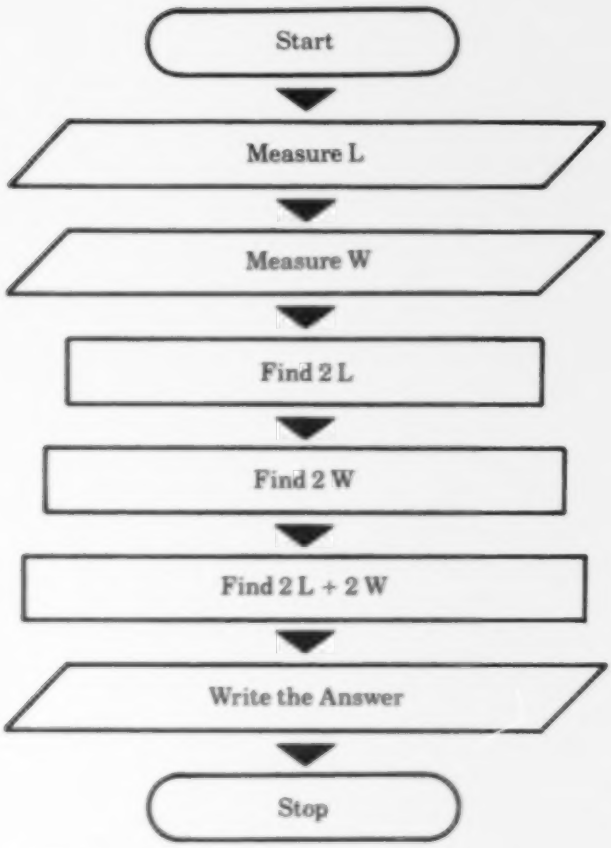
Before considering a problem in mathematics, construct a flowchart for a simple activity such as sharpening a pencil.



The students may construct flowcharts for various activities, such as:

- Brushing one's teeth
- Going shopping
- Preparing breakfast

Have the students develop a flowchart in class for some mathematical problem. For example, find the perimeter of any rectangle given its length, L, and its width, W.



b. Program

A *program* is a logical sequence of steps written in a programming language that can be followed by a computer. Such languages include BASIC, Pascal, and LOGO. The teacher should feel free to present a program in any acceptable computer language.

For example, to find the perimeter of a rectangle, we might write the following program in the BASIC language: (Note that * means multiply.)

```
10 INPUT L
20 INPUT W
30 LET P = 2*L + 2*W
40 PRINT P
50 END

RUN
```

The steps of this program are numbered 10 through 50 because the "control" component of the computer follows steps in numerical order. Suppose this program is entered into a computer, followed by the command RUN. The computer will respond with a "prompt" symbol such as a question mark, at which point we must enter a specific value for each required variable. For a length of 7 and a width of 3:

```
? 7
? 3
```

The values 7 and 3 are each placed in different memory cells, and these locations are named L and W, respectively.

Following the instruction given in line 30, the "arithmetic/logic" unit multiplies 7 by 2 to obtain 14; it multiplies 3 by 2 to obtain 6; and it adds 14 and 6 to obtain 20. This value is then stored in a memory cell at a location name P.

	MEMORY	MEMORY
	L	7
	W	3
	P	20
Output:		20

The students may construct flowcharts for simple mathematical activities, such as:

- Finding the area of a rectangle
- Finding the perimeter of a triangle
- Finding the total cost of 4 items at a supermarket
- Finding the average of 3 numbers

If computers are available, the students may try to write programs at this point or later in this unit.

4. Uses of Computers Today

Devote some time to a discussion of where computers are used in the world today. The students may display newspaper/magazine articles, or write reports on computer usage. Examples should include:

a. *School*

(1) Computer-assisted instruction
(2) Computerized report cards
(3) Simulations
(4) Computerized attendance
(5) Communication and other environmental interactions for students with handicapping conditions

b. *Home*

(1) Microwave ovens
(2) Bills (telephone, electric, etc.)
(3) Occupations of family members
(4) Computer games

(5) Communication and other environmental interactions for students with handicapping conditions

c. *Business/Industry*

(1) Inventories and records
(2) Accounting and payroll
(3) Reservations for planes, theaters, etc.
(4) Computer scans at supermarkets, libraries, etc.
(5) Gasoline filling stations

d. *Government*

(1) Internal Revenue Service
(2) Social Security
(3) Defense systems
(4) Space program
(5) Census

e. *Media*

(1) Science fiction in film and print
(2) Television documentaries

Other applications of computers, reports of new products, a history of computing devices, and so forth, may form the basis for a discussion or for student reports.

B. MATHEMATICAL OPERATIONS AND FUNCTIONS

This section may be taught without the use of a computer. If computers are available, however, they should be used by the students. In this case, the programming in part C may be integrated with the material in part B.

1. Operations Using Computer Symbols

The teacher should present the operational symbols used on the computer at the school. If no computer is available, present the symbols used herein from BASIC.

The students should realize that a programming language is used to give information to a computer. The words and symbols in the language are changed into electronic signals that open and close switches in the circuitry of the computer as it solves the problem. The answer we receive is coded back into the programming language.

	Arithmetic Expression	BASIC Expression
Addition:	A + B	A + B
Subtraction:	A – B	A – B
Multiplication:	AB, or A•B, or (A)(B)	A * B
Division:	$A \div B$, or A/B or $B\overline{)A}$	A/B
Raising to a power:	A^B	A↑B, or A ^ B, or A**B

(dependent upon version of BASIC)

For example, the following BASIC expressions are evaluated as shown:

BASIC Expression	Value of the Expression
10+2	12
10–2	8
10*2	20
10/2	5
10↑2	100

2. Evaluating Expressions in Computer Format

This material, if not integrated within the Number Concept strand, should be studied here as reinforcement of concepts presented earlier.

a. Order of operations

As a rule, the order of operations learned earlier has been incorporated into the internal structure of the computer. The students should evaluate numerical expressions which use operational symbols from a programming language.

Example: Evaluate each expression, (from the BASIC language).

1) 8 * 10 + 2 2) 8 + 10 * 2 3) (8 + 10) * 2
4) 10 – 4½ 5) (10 – 4)/2 6) 5 + 4 ↑ 2

Answer: 1) 82 2) 28 3) 36
4) 8 5) 3 6) 21

Example: 1) Evaluate 4 * (3 ↑ 2).
2) Evaluate (4 * 3) ↑ 2.

3) Write an expression having the *largest* possible numerical value using exactly one each of the numbers 2, 3, and 4, one each of the operations * and ↑, and one set of parentheses.

Answer: 1) 36 2) 144
3) Although 3 ↑ (4 * 2) = 6561, a creative student might search for an expression with a larger value.

Optional: If a computer is available, the students should learn how to use the device either as a computational tool or as a check of calculations performed with

paper and pencil. To do this, the students must be taught a simple PRINT-type command, and they must know how to translate standard operational symbols into the symbols of a programming language.

Example: 1) Write a statement to evaluate $5^2 - 4^2$ on a computer.

2) Find the computer output for the statement just written.

Answer: 1) PRINT 5 ↑ 2 – 4 ↑ 2
2) 9

Example: Jeannette wishes to check her answer for $\frac{2}{5} \times 65$ on a computer. How many of the following statements will give Jeannette the correct output?

1) PRINT 2/5 * 65
2) PRINT (2/5) * 65
3) PRINT 2 * 65/5
4) PRINT (2 * 65)/5

Answer: All four statements yield the correct output, 26. (Ask the students why this is true, and lead them into a discussion of the use of parentheses, the order of operations, and certain mathematical properties.)

b. Use of decimals in output

In solving any problem where the answer is not an integer, the computer will present the output in decimal notation.

Example: PRINT 3/6 *Output*: .5

Example: PRINT 15 * 1/4 *Output*: 3.75

The maximum number of digits shown in an output will vary from one computer to another. Some computers display 6-digit numerals, but most display 8-digit or 9-digit numerals. Because of this limitation, computers will round off decimals. In the following examples, imagine that the display is *restricted to eight digits*.

Example: PRINT 1/3 *Output*: .33333333

Example: PRINT 2/3 *Output*: .66666667

Rounding skills, as well as common fraction-to-decimal conversions, can be reviewed by asking the students to predict computer outputs.

Example: If a computer displays at most 8 digits, write the computer output for each statement:

1) PRINT .2468135790123456789
2) PRINT 5/8
3) PRINT 5/9

Output: 1) .24681358
2) .625
3). 55555556

Optional: If a computer is available, the students should enter statements as shown above and check their predictions. Operations involving fractions and mixed numbers can be performed on the computer, often leading to a discussion of grouping symbols and operational concepts. For example, to obtain an answer for $2\frac{1}{2} \times 3$, we enter:

PRINT (2 + 1/2) * 3

Why is the + sign needed? If the parentheses were not included above, could the correct answer be obtained? Why? Note that other statements are acceptable, such as:

PRINT 2.5*3

c. Use of scientific notation in output

To help us read numerals, we usually write commas. However, commas are *not* used when we input or output a number on a computer.

Arithmetic Expression	Computer Expression
56,295.3	56295.3
8,017,044	8017044

When a number becomes too large for a display, the computer uses a form of scientific notation, called *exponential form* or *E-form*. We learned that 2,654,000,000 = 2.654×10^9 in scientific notation. If a computer is restricted to 8 digits, it will display this 10-digit number as follows:

2.654 E 9

Here, the E represents a base of 10, and the 9 indicates an exponent. Just as scientific notation was limited in the Number Concept strand to values larger than or equal to one, E-form should be restricted to similar values in grade 7.

Example: What is the decimal form (standard notation) of 1.73×10^6?

Answer: 1,730,000

Example: What is the decimal form of 2.09 E 11?

Answer: 209,000,000,000

Example: If a computer display is restricted to 8 digits, find the output:

1) PRINT 20000 * 400000

2) PRINT 500 ↑ 3

Output: 1) 8 E 9
2) 1.25 E 8

3. Functions Used with Computers

a. Absolute value function, ABS (X)

Present the students with a number line in which two points, X and Y, correspond to the numbers 3 and 7 respectively. What is the distance between these points? Here, Y – X = 7 – 3 = 4.

Now imagine a number line with two points A and B. This time, however, the corresponding numbers are not given. How can we write a rule to find the distance between these points? Why is B – A not the correct rule? Why is A – B also incorrect? Think of any two numbers, say 2 and 9:

If A = 2 and B = 9, then B – A = 9 – 2 = 7. The distance is 7. But if A = 9 and B = 2, then B – A = 2 – 9 = – 7. Distance must be positive. Similar problems exist for the rule A – B.

Show how absolute value, seen in the Number Concept strand, can resolve this problem. Note that operations within absolute value symbols are performed first, just as operations within parentheses are performed first. Either |B – A| or | A – B | may be used to determine the distance.

If A = 2 and B = 9, then |B – A| = |9 – 2 | = |7| = 7. Also, |A – B| = 7.

If A = 9 and B = 2, then |B – A| = |2 – 9| = |–7| = 7. Also |A – B| = 7.

Show the students how the absolute value function is entered on the computer at school. If no computer is available, present the BASIC form of the absolute value function:

ABS (X)

Study problems that reinforce concepts and skills with signed numbers, distance, and algebraic expressions.

Example: Evaluate each computer expression.

1) ABS (28) 2) ABS (–93) 3) ABS (–2.56)
4) ABS (8 – 14) 5) ABS (–2 + 18) 6) ABS (–3 – 4)

Answer: 1) 28 2) 93 3) 2.56
4) 6 5) 16 6) 7

Example: For what value(s) of R will ABS (R – 4) equal zero?

Answer: 4

Example: For what value(s) of M will ABS (M – 2) equal 5?

Answer: 7 and –3

Example: Tell which of the following three statements is false, and find numerical values to demonstrate why.

(1) ABS (P) = ABS (–P)
(2) ABS (P – Q) = ABS (P) – ABS (Q)
(3) ABS (P – Q) = ABS (Q – P)

Answer: Choice (2) is false. If P = 5 and Q = 8, then ABS (5 – 8) = ABS (– 3) = 3, while ABS (5) – ABS (8) = 5 – 8 = –3.

b. Greatest integer function, INT (X)

A ballpoint pen costs $.40. How many of these pens can be purchased with $3?

$$.40 \overline{)3.00}\quad 7.5$$

Since it is impossible to purchase 7.5 pens, the answer is 7 pens. The integer 7 is the *greatest integer* contained in the number 7.5.

In BASIC, the greatest integer function is usually written as INT (X).

In the problem just stated, INT (7.5) = 7. In the writing of a full statement of the solution to the problem, INT (3.00/.40) = INT (7.5) = 7.

The definition of the greatest integer may be presented in cases:

1) If X is an integer, then INT (X) = X.

2) If X is not an integer, then INT (X) is the greatest integer that is "less than X."

Example: Evaluate each BASIC expression.

1) INT (15) 2) INT (0) 3) INT (3.9) 4) INT (4 + ½) 5) INT (0.3)

Answer: 1) 15 2) 0 3) 3 4) 4 5) 0

It is suggested that the greatest integer function be *restricted to positive numbers* on this level.

Inasmuch as parentheses are grouping symbols, all operations found within the parentheses are to be performed before the greatest integer is found.

INT (8.7 – 5.1) = INT (3.6) = 3
INT (2 * 3.9) = INT (7.8) = 7

On the other hand, if a single number is found within the parentheses, the greatest integer is found before performing any other operations.

INT (8.7) – 5.1 = 8 – 5.1 = 2.9
2 * INT (3.9) = 2 * 3 = 6

The teacher should provide practice involving order of operations with the greatest integer function, placing emphasis on positive numbers.

Example: Evaluate each computer expression.

1) INT (7.9 + 9.9) 2) INT (7/2) 3) INT (63 – 2.1)

Answer: 1) 17 2) 3 3) 60

Example: A textbook is 3.2 cm wide. How many of these textbooks can be stored on a shelf that is 88 cm wide?

INT (88/3.2) = INT (27.5) = 27

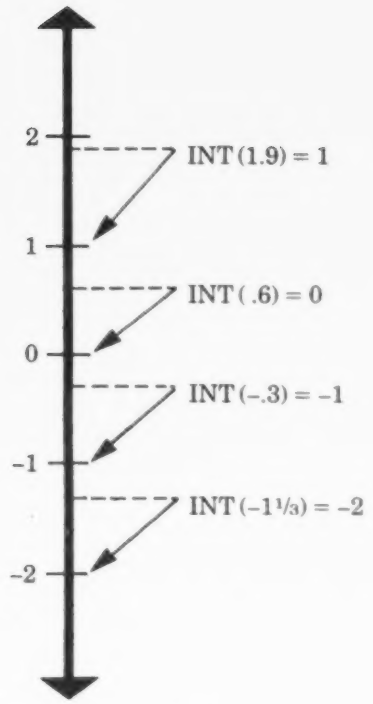
Answer: 27

Example: How many bottles of medicine, each holding 75 mL, can be filled from a large container that holds 1L of medicine?

INT (1L/75mL) = INT (1000/75) = INT (13. $\overline{3}$) = 13

Answer: 13

Optional: The students who solve more challenging problems may encounter negative values of X when using INT (X). A vertical number line is an aid when evaluating INT (X) for such negative values.



4. Capabilities and Limitations

Ask the students to list various capabilities and limitations of the computer. Possible items are given here.

- *Capabilities* of the computer:
  (1) Performs massive calculations with great speed
  (2) Makes logical decisions based on a comparison of data
  (3) Accurately stores and retrieves data
  (4) Displays information using graphics and sound
  (5) Acts as a word processor
  (6) Simulates events and situations

- *Limitations* of the computer:
  (1) Does not think
  (2) Has a limited memory
  (3) Produces errors due to rounding
  (4) Displays a limited number of digits

Other capabilities and limitations may be listed, but the students should sense that every user of computers must have the ability to tell if output from a computer is reasonable or not. In some cases, this requires estimation skills. In other cases, a knowledge of basic mathematics is essential. For example, a user would know that $^{20}/_{10} = 2$, although the computer may report that $^{20}/_{10} = 2.0000001$.

C. COMPUTER PROGRAMMING (OPTIONAL)

If computers are available in the school, the students should access them. This section of the computer strand is best understood if integrated with Part B of this unit.

In general, as a first step the teacher should refer to the manuals for the computers at hand, rather than use general texts. A logical approach to programming, beyond teaching students how to turn the system on and off and to load and run packaged software, might include these general steps:

1) Teach system commands.
2) Enter and run a prepared program.
3) Teach program commands.
4) Write a simple program.

1. Determine Computer Output of Given Programs

The students can benefit from activities in which they are asked to determine the computer output of given programs *without* running the programs.

Example: List the output that should be given when each of the following programs is run on the computer. (BASIC language)

Program A

```
10  LET A = 3
20  LET B = 4
30  LET C = 5
40  PRINT A/B
50  PRINT A - B + C
60  END
```

Output:

```
   .75
 4
```

Program B

```
10  FOR N = 1 TO 4
20  PRINT N, N*N
30  NEXT N
40  END
```

Output:

```
1       1
2       4
3       9
4       16
```

Example: List the output that should be given when the following program is run on the computer (Pascal language)

```
program     Square;
var N:      integer;

begin
for N: = 1 to 6 do
    begin
        write ln (N,'', N*N)
    end;
end.
```

Output:

```
1       1
2       4
3       9
4       16
5       25
6       36
```

2. Algorithms Used in Programming

The greatest integer function can be used to develop algorithms appropriate to grade 7 when the domain of X is restricted to positive numbers. Two such algorithms involve rounding procedures and tests for divisibility. These algorithms can be incorporated into programs.

Algorithm 1: Rounding to the nearest integer

Ask the students if INT (X) rounds positive values of X to the nearest integer. Select values for X to see why the answer is "no."

8.7 = 9 (to the nearest integer), but INT (8.7) = 8
.64 = 1 (to the nearest integer), but INT (.64) = 0
3.2 = 3 (to the nearest integer), and INT (3.2) = 3

In some cases, INT (X) gives the correct result, but in others it does not. Challenge the students to find some constant value C so that INT (X + C) always names the correct value of X rounded to the nearest integer. Eventually, present the rule for rounding and test it:

INT (X + .5)

If X = 8.7, then INT (X + .5) = INT (8.7 + .5) = INT (9.2) = 9

If X = .64, then INT (X + .5) = INT (.64 + .5) = INT (1.14) = 1

If X = 3.2, then INT (X + .5) = INT (3.2 + .5) = INT (3.7) = 3

Once the algorithm is understood, ask the students how a computer could be directed to round any positive number X to the nearest integer. The response should simply be:

PRINT INT (X + .5)

Algorithm 2: Rounding to the nearest ten

If the students ask how the computer rounds to the nearest ten, or if the teacher wishes to pursue this topic, either of the following algorithms may be presented and discussed:

INT (X/10 + .5) * 10

or

INT ((X + 5)/10) * 10

To understand how either or both of these algorithms work, select values for X and follow the steps taken by the computer. For example:

"83 rounded to the nearest ten is 80."

```
INT (X/10 + .5) * 10          or  INT ((X + 5)/10) * 10
INT (83/10 + .5) * 10         INT ((83 + 5)/10) * 10
INT (8.3 + .5) * 10           INT (88/10) * 10
INT (8.8) * 10                INT (8.8) * 10
   8          * 10                  8          * 10
  80                            80
```

In the next example, we round upward:

"56 rounded to the nearest ten is 60."

```
INT (X/10 + .5) * 10          or  INT ((X + 5)/10) * 10
INT (56/10 + .5) * 10         INT ((56 + 5)/10) * 10
INT (5.6 + .5) * 10           INT (61   /10) * 10
INT (6.1   ) * 10                INT (6.1   ) * 10
   6     * 10                 6      * 10
   60                            60
```

Similar algorithms exist for rounding positive numbers to the nearest hundred, nearest thousand, etc. The discovery of these algorithms is best treated as an activity for interested students.

Algorithm 3: Tests for divisibility

Although various algorithms for divisibility exist, limit discussion here to a solution involving the greatest integer function. Ask the students to evaluate N/2 and INT (N/2) for any whole numbers they choose.

```
              N/2          INT (N/2)
Let N = 26 26/2 = 13       INT (26/2) = INT (13) = 13
Let N = 7   7/2 = 3.5      INT (7/2) = INT (3.5) = 3
Let N = 74 74/2 = 37       INT (74/2) = INT (37) = 37
```

When does N/2 = INT (N/2)? The students should realize that this equality is true when N is an even number, or when "N is divisible by 2."

Similarly, if N is divisible by 2, then:

N/2 – INT (N/2) = 0

Ask the students to evaluate N/3 and INT (N/3) for any whole numbers chosen, leading to the discovery that "N is divisible by 3" if:

N/3 = INT (N/3)

or

N/3 – INT (N/3) = 0

This algorithm can be extended to tests for divisibility by 4, 5, 6, and so forth.

3. Write Original Programs

Student programs may include a listing of common fraction-to-decimal conversions, with the level of programming dependent upon the ability of the students. For example, consider a program to print conversions for 1/4, 2/4, 3/4, and 4/4. The levels of difficulty may vary from the simple to the complex in writing the program. Consider those variations starting with LET statements, progressing to READ/DATA statements, moving to FOR/NEXT loops, and finally including quotations. (It is recommended that the students *not* see statements of the form N = N + 1 at this level because of the confusion that may result as they begin to study the foundations of algebra.)

Program 1:

```
10 LET A = 1
20 LET B = 2
30 LET C = 3
40 LET D = 4
50 PRINT A/4
60 PRINT B/4
70 PRINT C/4
80 PRINT D/4
90 END
```

Program 2:

```
10 READ A, B, C, D
20 DATA 1, 2, 3, 4
30 PRINT A/4
40 PRINT B/4
50 PRINT C/4
60 PRINT D/4
70 END
```

Program 3:

```
10 FOR N = 1 TO 4
20 PRINT N/4
30 NEXT N
40 END
```

1, 2, and 3 Output for Programs:

```
.25
.5
.75
1
```

Program 4:

```
10 FOR N = 1 to 4
20 PRINT N; "/4 ="; N/4
30 NEXT N
40 END
```

Output for Program 4:

```
1/4 = .25
2/4 = .5
3/4 = .75
4/4 = 1
```

The use of a conditional statement, such as the IF-THEN statement in BASIC, can be related to the decision box used in flowcharting. As an example, the following program is used to see if a whole number is divisible by 3.

```
10 INPUT N
20 IF N/3 = INT (N/3) THEN 50
30 PRINT N; "IS NOT DIVISIBLE BY 3"
40 GO TO 60
50 PRINT N; "IS DIVISIBLE BY 3"
60 END
```

4. Additional Activities

The following list contains just a few of the additional activities which may be considered for grade 7:

- For any two given numbers, X and Y, calculate their sum, difference, product, and quotient. ($Y \neq 0$)
- Print your name ten times.
- Print a table of squares (and/or cubes) for all integers from 1 to 10.
- Find the distance between two points on the rational number line whose coordinates are given as input.
- Round any positive number that is given as input to the nearest integer, nearest ten, nearest hundred, etc.
- (1) Find the average of any four numbers given as input.
  (2) State this average to the nearest tenth.
- Print a multiplication table such as the 5-times table:

  $5 * 1 = 5$
  $5 * 2 = 10$
  $5 * 3 = 15$
  ...
  $5 * 10 = 50$

- Input the number of times a baseball player goes to bat, and the number of hits that the player gets. Then express the player's batting average in the usual notation, that is, as a decimal to the nearest thousandth (such as .298 or .345).
- Input the length and width of a rectangle, and have the output list the perimeter and area of the rectangle.
- For any whole number given as input, find all factors of the number.

Many other programs can be developed within content strands of this curriculum.

VII: GEOMETRY AND MEASUREMENT

Inasmuch as the study of geometry and measurement is a major strand in the K-6 *Syllabus*, the students will be familiar with a number of concepts presented here in grade 7. At this stage, however, the students should begin to understand and use proper terminology and symbols, definitions, and formulas.

The systems of measure found in sections B and C of this unit might have been studied at an earlier time and used in applications throughout the curriculum. If so, the teacher may wish to integrate the measurement of geometric figures from section D with the development of geometry found in section A.

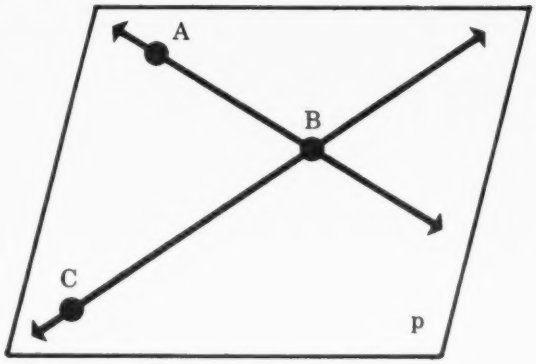
A. GEOMETRY

1. Undefined Terms: Point, Line, and Plane

Geometry, like any mathematical system, begins with *undefined terms*. If the meaning of certain basic words can be accepted intuitively, then the other terms can be defined in terms of these basic words. Some undefined terms are *point, straight line,* and *plane*.

- A *point* has no dimensions but possesses the single property of position.
- A *curve* is a set of points represented by a figure drawn without lifting the pencil point from the page.
- A straight *line* is a curve which is a set of points extending infinitely in opposite directions, thus possessing the single property of infinite length. A *line* is determined by two points. The term *line* is understood to mean straight line.
- A *plane* can be thought of as a flat surface, possessing the properties of infinite length and infinite width. Three points, not in the same straight line, determine a plane.

For example, in the plane p, determined by points A, B, and C, two lines are illustrated:



Line AB, symbolized as $\overleftrightarrow{AB}$ or $\overleftrightarrow{BA}$;

Line BC, symbolized as $\overleftrightarrow{BC}$ or $\overleftrightarrow{CB}$.

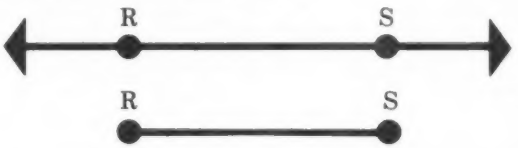
Points A and C determine a

third line, namely $\overleftrightarrow{AC}$ or $\overleftrightarrow{CA}$.

2. Line Segment, Ray, and Pairs of Lines

A *line segment*, also called a *segment*, is a finite portion of a line and has two endpoints. A *ray* is an infinite portion of a line and has only one endpoint. In the naming of a ray, the endpoint is stated first.

Given any line such as $\overleftrightarrow{RS}$, the students should be able to identify segments and rays.



segment RS, symbolized as

$\overline{RS}$ or $\overline{SR}$

ray RS, symbolized

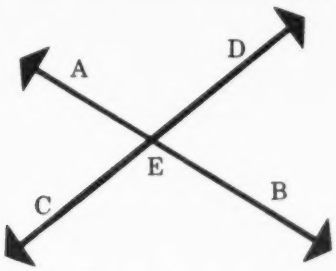
as $\overrightarrow{RS}$

ray SR, symbolized

as $\overrightarrow{SR}$

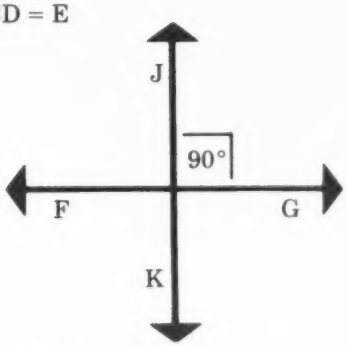
Consider pairs of lines in a plane. Two lines are called *intersecting* if and only if they meet at exactly one point. If the lines intersect to form right angles (see the next section), the lines are called *perpendicular*. Two lines in the same plane that never meet are called *parallel lines*.

The teacher may introduce symbols to represent these relations.



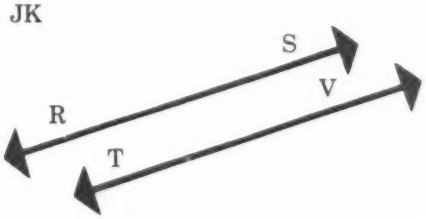
$\overleftrightarrow{AB}$ intersects $\overleftrightarrow{CD}$ at E.

$\overleftrightarrow{AB} \cap \overleftrightarrow{CD} = E$



$\overleftrightarrow{FG}$ is perpendicular to $\overleftrightarrow{JK}$.

$\overleftrightarrow{FG} \perp \overleftrightarrow{JK}$



$\overleftrightarrow{RS}$ is parallel to $\overleftrightarrow{TV}$.

$\overleftrightarrow{RS} \parallel \overleftrightarrow{TV}$

Example: In the diagram, points A, N, and T determine a plane because they do not lie in a single line. Using these points only, name all possible lines, segments, and rays.

Answer:

Three lines: $\overleftrightarrow{AN}$ (or $\overleftrightarrow{NA}$); $\overleftrightarrow{NT}$ (or $\overleftrightarrow{TN}$); $\overleftrightarrow{AT}$ (or $\overleftrightarrow{TA}$).

Three segments: $\overline{AN}$ (or $\overline{NA}$); $\overline{NT}$ (or $\overline{TN}$); $\overline{AT}$ (or $\overline{TA}$).

Six rays: $\overrightarrow{AN}$; $\overrightarrow{NA}$; $\overrightarrow{NT}$; $\overrightarrow{TN}$; $\overrightarrow{AT}$; $\overrightarrow{TA}$.

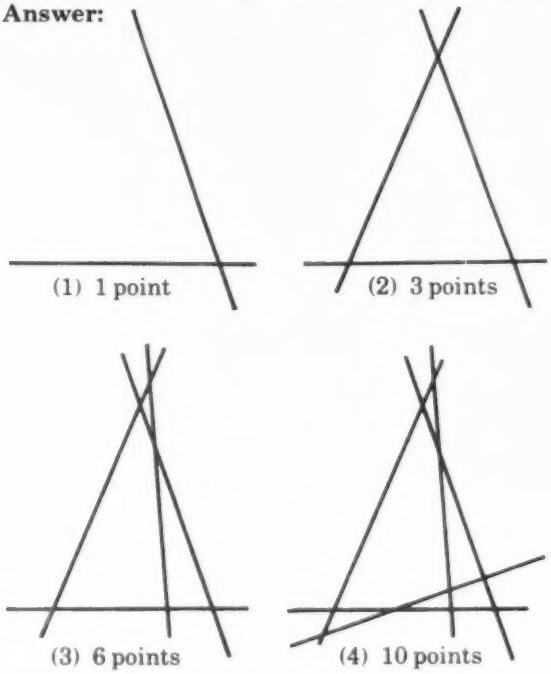
Example: $\overline{ABC}$ names a segment in which point B lies between points A and C. Which is *not* another name for $\overline{ABC}$?

(1) $\overline{AC}$ (2) $\overline{CA}$ (3) $\overline{BC}$ (4) $\overline{CBA}$

Answer: (3) $\overline{BC}$

Example: In a plane, find the greatest number of points of intersection using: (1) 2 lines (2) 3 lines (3) 4 lines (4) 5 lines. Can you find a pattern to extend the question to 6 or more lines?

Answer:



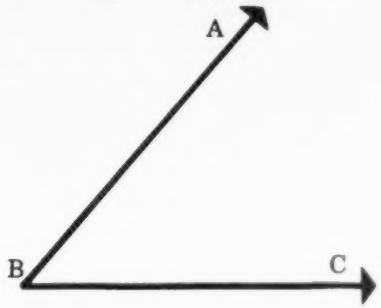
The number of points corresponds to the triangular numbers.

3. Angles

a. Definition and symbolism

An *angle* is the set of points contained in two rays that have a common endpoint.

In angle ABC, symbolized as ∠ ABC, the rays BA and BC are called the *sides* of the angle, and the common endpoint B is called the *vertex* of the angle. Here, ∠ ABC may also be identified as ∠ CBA or as ∠ B. Since a single letter cannot be used in ambiguous cases, stress the use of three letters when naming angles.



b. Degree measure and classification

A complete rotation measures 360 degrees. Therefore a degree, written 1°, is $\frac{1}{360}$ of a rotation. Classify angles according to their measures:

- An *acute angle* has a measure less than 90°.
- A *right angle* ($\frac{1}{4}$ rotation) has a measure of 90°.
- An *obtuse angle* has a measure greater than 90° but less than 180°.
- A *straight angle* ($\frac{1}{2}$ rotation) has a measure of 180°.

(**Optional:** A *reflex angle* has a measure greater than 180° but less than 360°).

c. Protractor

The students should become familiar with the use of the protractor, and know how to measure angles and how to draw angles using this tool. Most students will handle a semicircular protractor containing two scales. The typical error of using the wrong scale when working with an acute or an obtuse angle can be avoided by having the students first answer the question, "Is the angle acute and less than 90°, or is it obtuse and greater than 90°?"

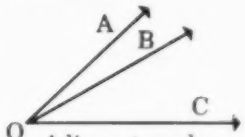
Example: Draw any triangle. Use a protractor to measure its three angles. Find the sum of these angle measures.

Answer: 180° (The teacher may wish to list individual answers on the chalkboard before citing the correct solution, leading to a discussion of precision in measurement.)

d. Pairs of angles: complementary, supplementary, adjacent, and vertical

Complementary angles are two angles whose sum is equal to a right angle. That is, the sum of the measures of the two angles is 90°. *Supplementary* angles are two angles whose sum is equal to a straight angle. That is, the sum of the measures of the two angles is 180°.

Adjacent angles, such as ∠ AOB and ∠ BOC in the diagram share a common ray (here, OB) and a common vertex (here, point O) and have no interior points in common. Complementary and supplementary angles need not be adjacent.

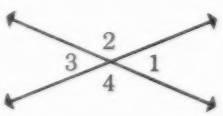


Adjacent angles

Complementary angles

Supplementary angles

When two lines intersect, the nonadjacent pairs of angles formed are called *vertical* angles. The students may experiment with protractors to discover that:

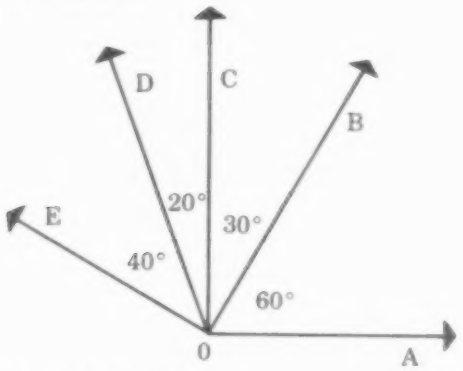


(1) Vertical angles, such as 1 and 3, are equal in measure.

(2) The sum of the angle measures about a point is 360°.

(3) Four pairs of supplementary angles are formed (∠ 1 and ∠ 2; ∠ 2 and ∠ 3; ∠ 3 and ∠ 4; ∠ 4 and ∠ 1).

Example: In the accompanying diagram, all angle measures are given.



a) What angle has the same measure as ∠ COE?

b) What angle is complementary to ∠ COE?

c) What angle has the same measure as ∠ AOC?

d) What ray is perpendicular to just $\overleftrightarrow{OE}$?

e) How many different angles are found in the diagram?

Answer: a) ∠ AOB b) ∠ BOC c) ∠ BOE
d) $\overrightarrow{OB}$ e) 10

4. Simple Closed Curves

A *closed curve* is a continuous curve drawn so that it ends where it began. A *simple closed curve* is one that does not intersect itself.

SIMPLE CLOSED CURVE

NON-SIMPLE CLOSED CURVE

Demonstrate that every simple closed curve separates the plane into three sets of points:

(1) the curve itself;

(2) the interior of the curve, which is called a *region*; and

(3) the exterior of the curve.

5. Polygons

a. Definition and types of polygons

A *polygon* is a simple closed curve formed by line segments. What is the least number of sides that a polygon could have?

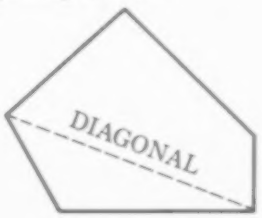
Mention the use of prefixes in identifying polygons.

Three sides:	triangle	Six sides:	hexagon
Four sides:	quadrilateral	Eight sides:	octagon
Five sides:	pentagon	Ten sides:	decagon

Optional:

Seven sides:	heptagon	Twelve sides:	dodecagon
Nine sides:	nonagon	Twenty sides:	icosagon

Example: A line segment that joins two nonadjacent vertices of a polygon is called a *diagonal*. How many diagonals does a pentagon have?



Answer: 5

Example: Complete the table shown below by drawing a series of polygons and their diagonals.

Number of sides	3	4	5	6	7	8
Number of diagonals						

Find a pattern to extend the table to polygons of 9 or more sides.

Answer:

Number of sides	3	4	5	6	7	8
Number of diagonals	0	2	5	9	14	20

Pattern: +2 +3 +4 +5 +6

b. Triangles

(1) Classification by angles

- An *acute triangle* contains three acute angles.
- A *right triangle* contains one right angle.
- An *obtuse triangle* contains one obtuse angle.
- An *equiangular triangle* contains three angles of equal measure (60° each).

(2) Classification by sides

- An *isosceles triangle* has two sides of equal measure (or equal length).
- An *equilateral triangle* has three sides of equal measure.
- A *scalene triangle* has no sides of equal measure.

Experimentation involving measurement with protractor and ruler can lead to the discovery of properties of triangles on an informal basis. For example:

Property 1: Base angles of an isosceles triangle are equal in measure;

Property 2: The sum of the angle measures of a triangle is 180°;

Property 3: The greatest angle of a triangle is opposite the longest side; etc.

Once these concepts are understood, they can be incorporated into problems.

Example: If two angles of a triangle measure 40° and 70°, the triangle must be:

(a) obtuse (b) right (c) isosceles (d) equilateral

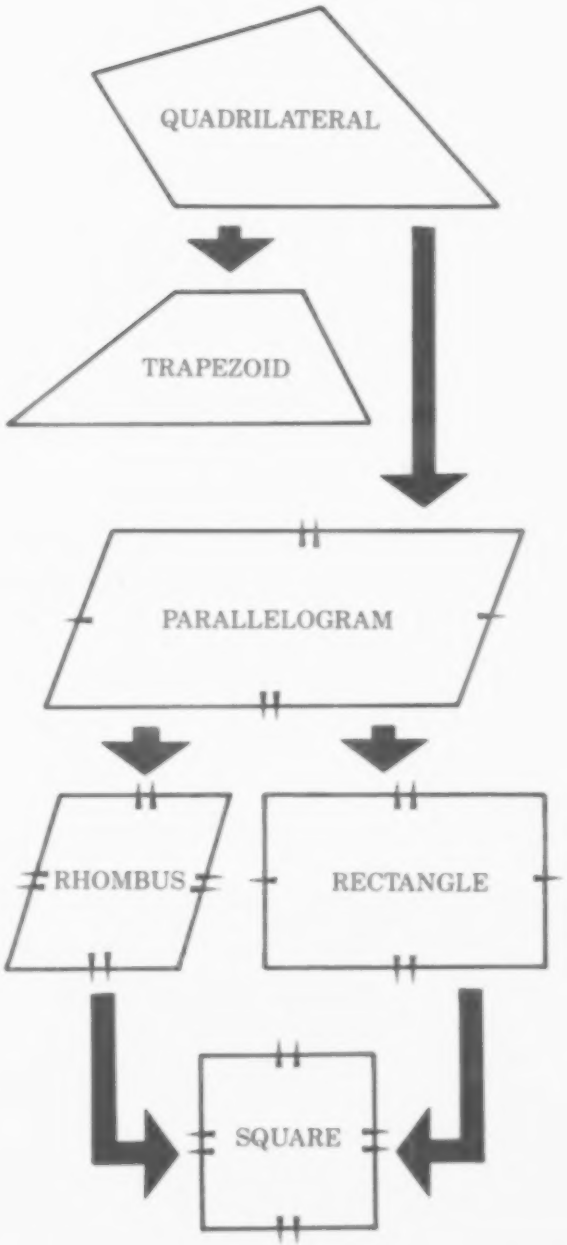
Answer: (c) isosceles

Example: In △ ABC, ∠ A measures 50° and ∠ B measures 70°. Which side of △ ABC is the shortest side?

Answer: $\overline{BC}$ or $\overline{CB}$

c. Quadrilaterals

A *quadrilateral* is any polygon of four sides. Experimentation and measurement should again play a role in helping the students to understand the types of quadrilaterals, their definitions, and some basic properties. A diagram can be used to organize the family of quadrilaterals.



(1) A *trapezoid* is a quadrilateral with only one pair of opposite sides parallel.

(2) A *parallelogram* is a quadrilateral with both pairs of opposite sides parallel. Experimentation involving tools of measure or paper folding can lead to informal observations:

- Opposite sides and opposite angles of a parallelogram are equal in measure. (Sides of equal length are indicated by using the same markings in the diagram above, such as | and ||.) Cut a parallelogram

along a diagonal and fit one half over the other.

- Diagonals of a parallelogram bisect each other.
- Consecutive angles of a parallelogram are supplementary.

(3) A *rectangle* is a parallelogram whose angles are right angles. Discuss the more concise definition: A rectangle is a parallelogram one of whose angles is a right angle.

(4) A *rhombus* is a parallelogram whose sides are equal in length. To be more concise: A rhombus is a parallelogram having two adjacent sides equal in length.

(5) A *square* can be defined in two ways (notice the diagram): A square is a rectangle whose sides are equal in length, or a square is a rhombus whose angles are right angles. More concise definitions can be stated.

d. Problem solving

Problem-solving techniques can be applied to relatively simple situations and extended by alterations in the question and/or the figure.

Example: In Figure A, find the total number of squares.

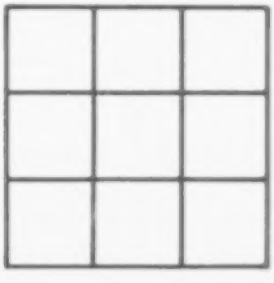


FIGURE A

Answer: 14

Example: In Figure A, find the total number of rectangles.

Answer: 36 (Remember, every square is a rectangle.)

Example: In Figure B, find the total number of:

(a) squares (b) rectangles (c) parallelograms

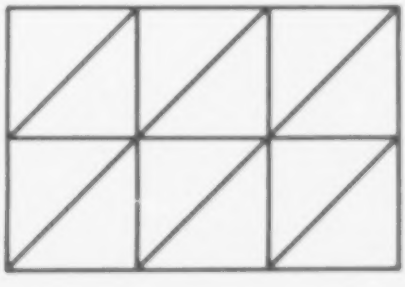


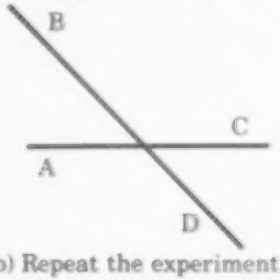
FIGURE B

Answer: (a) 8

(b) 18 (Every square is a rectangle.)

(c) 28 (Every rectangle is a parallelogram.)

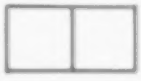
Example: (a) With a ruler, draw two segments called AC and BD so that they bisect each other. Connect the endpoints to form quadrilateral ABCD. Give the most specific name possible for this quadrilateral.



Example: (b) Repeat the experiment described above, but let AC and BD be the same length. Give the specific name for quadrilateral ABCD.

Answer: (a) parallelogram
(b) rectangle

Example: A domino consists of 2 squares having a common side. There is only one possible shape for a domino, because every domino can be turned and made to fit over the same simple shape.



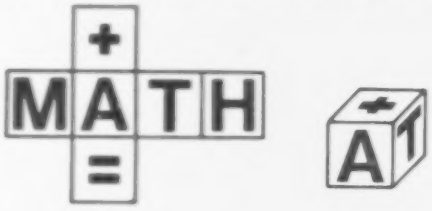
A triomino consists of 3 squares joined by common sides. How many different shaped triominoes exist?

Answer: 2

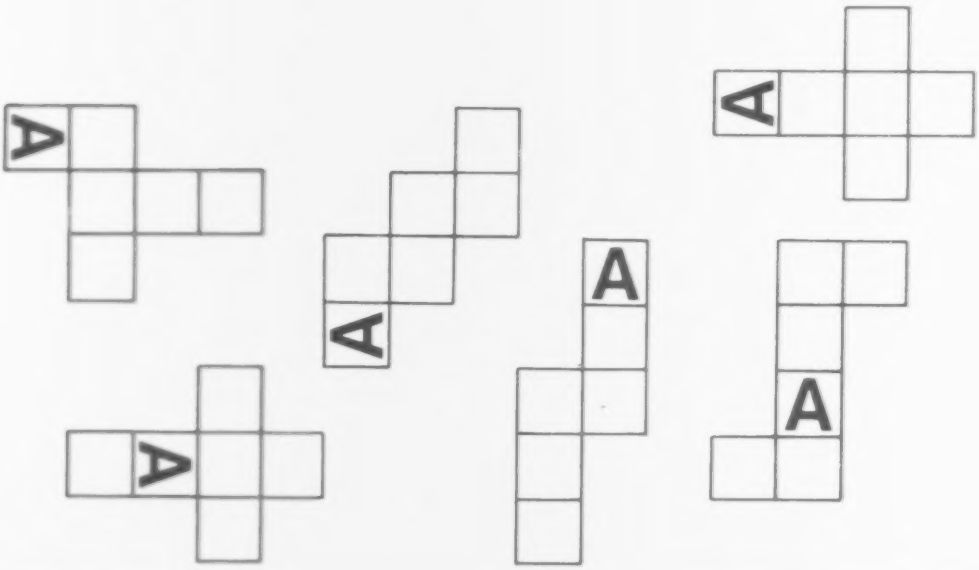
Finding the number of different shaped polyominoes of 4 squares (there are 5) and of 5 squares (there are 12) can pose a real challenge for a class.

Some polygons, such as the hexominoes in the following example, can be cut and pasted to give the students practice in spatial relationships.

Example: If the polygon at the bottom left were cut out and folded, it would form a unit cube. The letters in the word MATH would appear on four faces or surfaces of the cube, and the + and = signs would appear on the top and bottom faces.



For each pattern shown below, fill in the missing letters and symbols so that, when folded, the cube formed will be identical to the original cube shown above.



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6. Circles

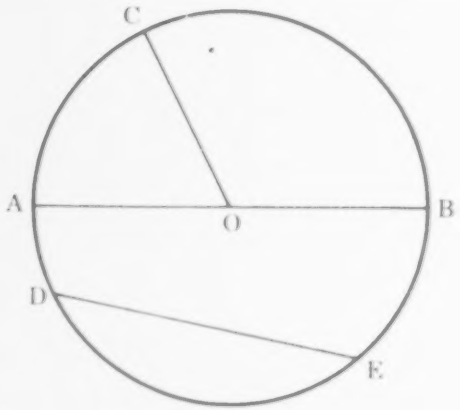
A *circle* is the set of all points in a plane which are at a fixed distance from a fixed point. The fixed distance is called the *radius*, and the fixed point is called the *center*. A *chord* is a line segment whose endpoints are on the circle; the longest chord in a circle is called a *diameter*. An *arc* is that part of a circle which is between two points on the circle, including these two points. An arc that is half a circle is a *semicircle*.

Example: Circle O is named by its center, point O. Use the diagram of circle O to name:

(1) all radii (2) all diameters (3) all chords

(4) the two arcs that are joined to form arc ADE

(5) all semicircles



Answer: (1) $\overline{OA}$, $\overline{OB}$, $\overline{OC}$

(2) $\overline{AB}$ or $\overline{AOB}$

(3) $\overline{AB}$, $\overline{DE}$

(4) arc AD and arc DE, written in symbols as $\overset{\frown}{AD}$ and $\overset{\frown}{DE}$

(5) arc $\overset{\frown}{ACB}$ and arc $\overset{\frown}{ADEB}$ (To distinguish one semicircle from another, include one or more points on the arc between their endpoints when naming them.)

B. METRIC SYSTEM

Measures from the metric system should be used throughout the curriculum in applications and in problem-solving situations.

1. Basic Units of Length, Mass, and Capacity

Review the basic units for length, mass or weight, and capacity or liquid measure. When using the metric system, conversions should not be made to units in the customary system. It is appropriate, however, to cite some approximations between measures in the two systems for purposes of introduction or estimation.

Length: the *meter* (m) is slightly longer than a yard.

Mass or Weight: the *gram* (g) is about the weight of a paper clip.

Capacity or Liquid Measure: the *liter* (L) is slightly more than a quart.

2. Prefixes and Symbols

In the metric system, a prefix is placed in front of the unit term to name a larger or smaller measure. Each prefix indicates a power of 10 and is represented by a symbol. Although other prefixes exist in the metric system, the following six prefixes should be known:

Prefix	*Example*
kilo (k) means 1000	1 kilometer (km = 1000 meters)
hecto (h) means 100	1 hectoliter (hL) = 100 liters
deka (da) means 10	1 dekagram (dag) = 10 grams
deci (d) means $^1/_{10}$ or .1	1 deciliter (dL) = 0.1 liter
centi (c) means $^1/_{100}$ or .01	1 centimeter (cm) = 0.01 meter
milli (m) means $^1/_{1000}$ or .001	1 milligram (mg) = 0.001 gram

The most commonly used prefixes are *kilo-*, *centi-*, and *milli-*.

3. Equivalent Measures within the Metric System

The students should have some sense of the proper metric measures to use in given situations. A person's height is usually measured in centimeters; the distance between towns is measured in kilometers.

Have the students find the missing metric measures in sentences.

Example: My bicycle weighs 8_____.

Answer: kilograms, or kg

Beyond this sense of measure, the students should be able to convert from one unit to another within the metric system. Procedures for conversion can include multiplication or division by powers of 10, or substitution by the prefix meaning.

Example: Give the numeral that will make each statement true.

a) 1L = _____mL b) 1 cm = _____ mm

c) 8000g = _____ kg d) 250 cm = _____ m

e) 4.6 km = _____ m f) 500 mg = _____ g

Answer: a) 1000 b) 10 c) 8

d) 2.5 e) 4600 f) 0.5

Extend metric measures to include the *metric ton*:

1 metric ton = 1000 kilograms

Example: A van can haul 8400 kg of material to a construction site. If the van makes three trips each day carrying a full load, how many metric tons of material does it haul daily?

Answer: 25.2 metric tons

4. Other Measures: Temperature, time, area, and volume

- Temperature: *degree Celsius* (°C)

  The students should know basic readings: the temperature of the freezing point of water is 0°C; the temperature of the boiling point of water is 100°C; normal body temperature is 37°C. Outdoor temperature readings also give the students a sense of the Celsius scale. There is little sense in teaching the conversion of readings from Celsius to Fahrenheit and vice-versa.

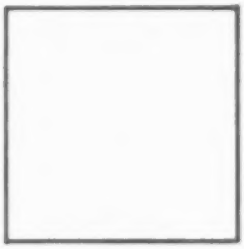
If a metric-customary conversion is necessary, however, for *any* measures in a problem-solving situation, the students should be provided with conversion factors obtained from tables, charts, or formulas.

- Time: *second* (s)

  In some problem-solving situations, a rate may be recorded in meters per second (m/s).

- Area: *Square meter* (m^2)

  Area measure is based on square units. This topic should be studied with the measurement of geometric figures found in section D of this strand. The students should know how to find equivalent square measures for basic metric units. For example, a square that measures one meter on each side has an area of one square meter. This same square measures 100 centimeters on every side.



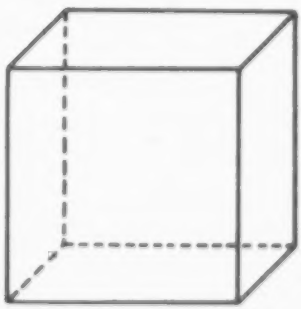
Since 1m = 100cm

$1m^2 = 10\,000cm^2$

In addition to the common area units of the square meter (m^2), the square centimeter (cm^2), and the square millimeter (mm^2), introduce the *hectare* (ha). A square that measures 100 meters on every side has an area of one hectare which is equal to 10 000 square meters. A hectare is used primarily as a measure of land, and is slightly less than 2.5 acres.

- Volume: *Cubic meter* (m^3)

  Volume is based on cubic units. This topic should be studied with the volume of geometric solids found in section D of this strand. The students should know how to find equivalent cubic measures for basic metric units.



Since 1m = 100cm

$1m^3 = 1\,000\,000cm^3$

The common metric units of volume include the cubic meter (m^3), the cubic centimeter (cm^3), and the cubic millimeter (mm^3). Although hospitals and science labs sometimes use cc as an abbreviation for cubic centimeter, the students should realize that cm^3 is the preferred symbol. Since a container whose volume is one cubic centimeter can hold one milliliter of water, the students might experience the interchange of these terms in a science experiment, that is, $1\ cm^3 = 1$ mL.

C. CUSTOMARY SYSTEM OF MEASURE

Although the use of customary units may be decreasing in our country, some familiarity with measurements and conversions within the customary system will be needed. If the students have not learned to measure in customary units, they should measure with inches (to the nearest $\frac{1}{16}$ inch), feet, yards, cups, pints, quarts, gallons, ounces, and pounds. Make measurement as activity-oriented as possible.

Discuss the units of area, including the square inch (sq. in.), square foot (sq. ft.), square yard (sq. yd.), and acre. An acre is a measure of land equal to 4840 square yards, or just slightly less than the size of a football field. Cubic units to be discussed include the cubic inch (cu. in.), cubic foot (cu. ft.), and cubic yard (cu. yd.).

The following equivalents should also be discussed:

1 foot = 12 inches	1 gallon = 4 quarts
1 yard = 3 feet = 36 inches	1 quart = 2 pints = 4 cups
1 mile = 5,280 feet	1 pound = 16 ounces (Avoirdupois)
1 sq. ft. = 144 sq. in.	1 ton = 2,000 pounds
1 cu. ft. = 1,728 cu. in.	1 minute = 60 seconds
1 cu. yd. = 27 cu. ft.	1 hour = 60 minutes

Conversions should be required only within the system, but some metric – customary comparisons can be made to gain a general sense of a measure. For example, an acre is about 0.4 hectare, and a mile is about 1.6 kilometers.

Customary units should be used occasionally in problems throughout the curriculum. Denominate numbers (numbers whose units represent units of measure, i.e., 2 meters, 4 seconds, 8 miles) should be studied.

Example: The Girl Scouts left camp at 8:50 a.m. and hiked for 1 hour and 25 minutes to reach Silver Lake. At what time did they reach Silver Lake?

Answer: 10:15 a.m.

D. MEASUREMENT OF GEOMETRIC FIGURES

1. Perimeter

a. Polygons

The students should realize that the distance between two points, unless otherwise noted, is the length of a line segment whose endpoints are those points. Relate this concept to a ruler and a number line.

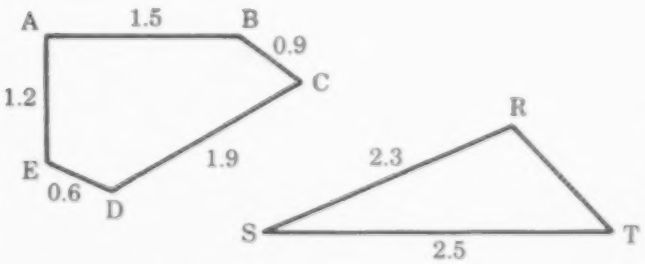
Use examples such as measuring the distance between opposite walls in a classroom, or measuring a person's height, to reach the following conclusion intuitively: the shortest distance between two parallel lines, two parallel planes, a point and a line, and a point and plane is always the perpendicular distance.

The *perimeter of a polygon* is the sum of the lengths of the sides of the polygon. Use this definition, rather than specific formulas, when treating perimeter. The students should measure perimeters of objects such as floor tiles or the chalkboard. The students may find the perimeter of *any* polygon.

Example: If one side of an equilateral triangle measures 5cm, find the perimeter of the triangle.

Answer: 15 cm

Example: The lengths of the sides of the polygons are given. If polygon ABCDE and triangle RST are equal in perimeter, find the length of $\overline{RT}$. (The length of $\overline{RT}$ may also be represented as RT.)



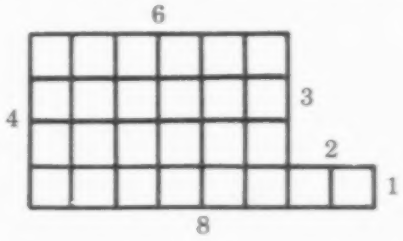
Answer: 1.3 cm

Example: If the perimeter of a rhombus is 6 feet, what is the length of one of its sides?

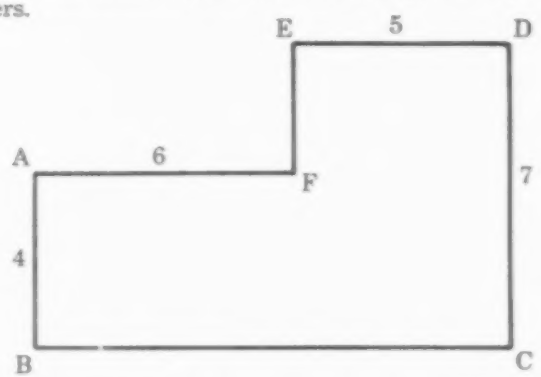
Answer: 1½ feet, or 1′6″, or 18 inches

Draw a polygon resembling an "L-shaped room" on graph paper. Note the relationship of the lengths of parallel segments. In the figure 6 + 2 = 8, and 1 + 3 = 4.

Observe that all angles in the polygon are right angles, and that the perimeter is 4 + 6 + 3 + 2 + 1 + 8 = 24.



Without the use of graph paper, study similar polygons consisting of two or more rectangles so that all angles are limited to right angles. Find lengths and perimeters.



Example: In polygon ABCDEF, let AB = 4, CD = 7, DE = 5, and FA = 6. Assume right angles.

(a) Find BC.

(b) Find EF.

(c) Find the perimeter of the polygon.

Answer: (a) BC = 6 + 5 = 11 (b) EF = 7 – 4 = 3 (c) 36

(Note that polygon ABCDEF has the same perimeter as a 7 by 11 rectangle. Why? Can a generalization be made?)

b. Circles: circumference

Using string or tape measures, let the students measure circular shaped objects such as the tops of tin cans, waste baskets, or cups. For each circle, measure the diameter d and the *circumference* C, that is, the length of the circle itself or its "perimeter." Allow the students to discover that C/d is a constant value. Some students will recognize that the ratio C/d is slightly more than 3; others will find a more precise value.

Define the ratio of the circumference of a circle to its diameter as a constant called *pi* (written in symbols as

π). Pi is an irrational number; it can never be stated precisely as a rational number or as a repeating decimal. To perform computations with π, rational number approximations are used:

3.14 or $^{22}/_{7}$ or 3.1416

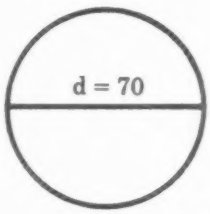
From the ratio $^{C}/_{d} = \pi$, show that $C = \pi d$.

Since the diameter is equal to twice the radius, or $d = 2r$, the formula $C = 2\pi r$ may also be used.

The students should understand that different approximations for π result in different approximations for the circumference.

Example:

If the diameter of a circle is 70mm, find its circumference.



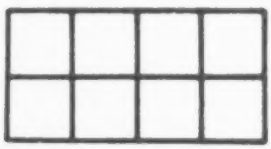
(Use $\pi = {}^{22}/_{7}$)	(Use $\pi = 3.14$)
$C = \pi d$	$C = \pi d$
$C = {}^{22}/_{7} \times 70$	$C = 3.14 \times 70$
$C = 220$	$C = 219.80$

Answer: 220 mm or 219.8 mm

2. Area

a. Quadrilaterals

(1) Area is a measure of a region expressed in square units. In earlier grades, the students used geoboards, dot paper, and/or graph paper to develop a formula for the area of a *rectangle*.



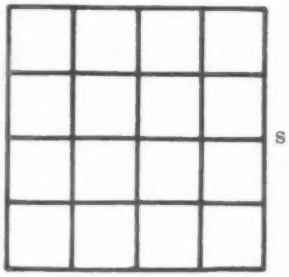
Using length l and width w *to find area A*: $A = lw$

Using base b and height h *to find area A*: $A = bh$

In either case, given the dimensions of a rectangle as 4 cm by 2 cm, its area is a product, 4 cm × 2 cm = 8 cm². The rectangle contains 8 units; each is one square centimeter in measure.

The area of every quadrilateral studied in section VII.A.5c can be described by a formula which is essentially "the product of a base and a height." Graph paper and diagrams serve as aids in deriving the formulas.

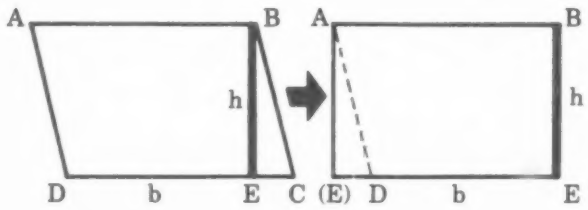
(2) A *square* is a rectangle. Therefore, its *area* = bh. But the base and height each equal side s, leading to the formula:



$A = s^2$

For example, if s = 8 inches, $A = s^2 = 64$ sq. in.

(3) In a parallelogram, the height h is a segment drawn perpendicular to the base b. In the diagram below, BE = h and DC = b. By the shifting of BEC along DC so that BC and AD coincide, the parallelogram is converted to a rectangle with base b and height h.



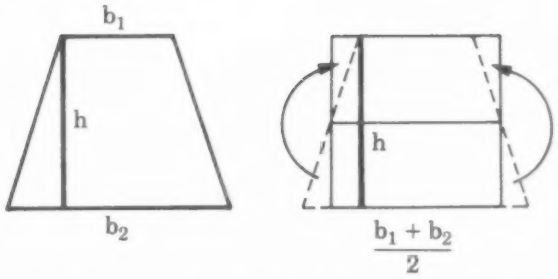
Since every parallelogram can be converted to a rectangle, the formula for the area A is:

$A = bh$

(4) A *rhombus* is a parallelogram. Therefore:

$A = bh$

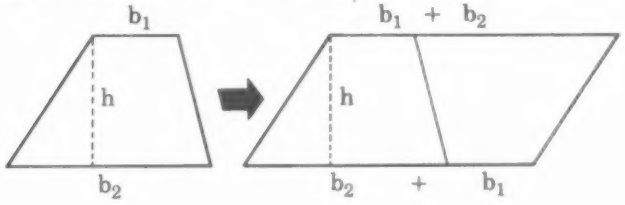
Optional: In a *trapezoid*, bases b_1 and b_2 are different lengths, and the height is given as h. Find the midpoint of each of the two sides that are not parallel. The segment joining these midpoints has a length that is the "average" of the two bases, namely $(b_1 + b_2) \times {}^{1}/_{2}$. By the shifting of two triangles as shown in the diagram below, the trapezoid is converted to a rectangle where "the base" is actually the average of two bases and the height is h.



Therefore, to find the area A of a trapezoid, use the formula:

$$A = \frac{b_1 + b_2}{2} \times h$$

Optional: An alternate approach involves the rotation of the trapezoid so that two trapezoids of the same size and shape are joined to form a parallelogram whose base is $(b_1 + b_2)$ and whose height is h.



Since the area of the parallelogram is $(b_1 + b_2)\,h$, and since the area of the trapezoid is half the area of the parallelogram,

$A = \frac{1}{2}(b_1 + b_2)\,h$

Standard area problems should include computation with whole numbers, decimals, and fractions. Problem-solving situations should be studied.

Example: Find the area of each quadrilateral.

(a) A parallelogram where b = 1.6 cm and h = 1.25 cm

(b) A square where side s = $3\frac{1}{2}$ feet

Answer: (a) $2cm^2$ (b) 12 ¼ sq. ft. or 1764 sq. in.

Example: The area of a rectangular rug is 60 square feet.

(a) If its length is 10 feet, find its width in feet.

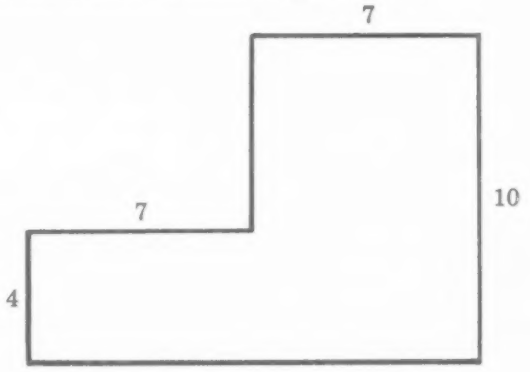
(b) How many feet of fringe are needed to make a border around the perimeter of the rug?

Answer: (a) 6 (b) 32

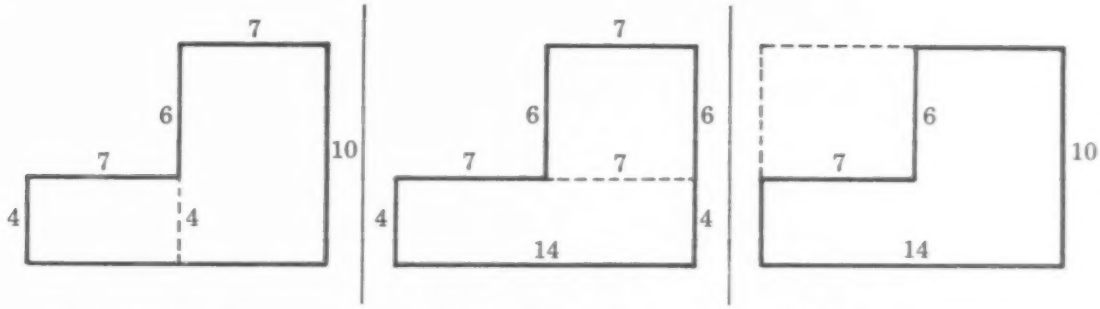
Example: Mr. Cautela has 24 feet of fencing, in one-foot lengths. He plans to fence off a rectangularly shaped garden in his back yard. Find the dimensions of the garden with the largest possible area, and state the area.

Answer: The dimensions are 6 feet by 6 feet (a square is a rectangle), and its area is 36 square feet.

Example: Find the area of the polygon whose sides are measured in meters. Assume right angles.



(The students should first find the lengths of two sides not given: 10 – 4 = 6, and 7 + 7 = 14. The problem then becomes one of adding or subtracting the areas of two rectangles.)



Area I = 7×4 = 28	Area I = 4×14 = 56	Area I = 14×10 = 140
Area II = 7×10 = +70	Area II = 6×7 = +42	Area II = 6×7 = –42
Total Area = 98	Total Area = 98	Total Area = 98

Answer: 98m²

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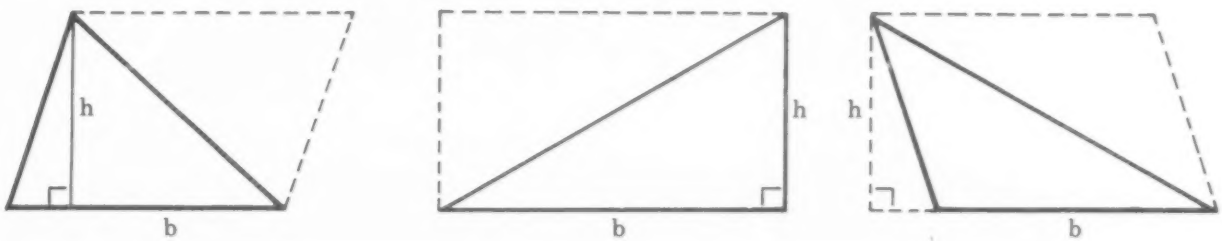
b. Triangles

It can be shown that the area of a triangle with base b and height h is always one-half of the area of a parallelogram with the same base and height.



Therefore for a triangle:

$A = \frac{1}{2}bh$

c. Circles

Introduce the formula for the area A of a circle:

$$A = \pi r^2$$

Use $\pi = 3.14$, or $\pi = \frac{22}{7}$, to find rational approximations of the area.

Example: Find the area of a circle whose radius is 14 yards. (Use $\pi = \frac{22}{7}$)

Answer: 616 sq. yd.

Example: Find the area of a circle whose diameter is 20 cm. (Use $\pi = 3.14$)

Answer: 314 cm²

3. Volume

A *geometric solid* is any closed surface that completely encloses a portion of space. *Volume* is a measure of the interior of the solid expressed in cubic units.

In grade 7, volume is limited to two solid figures: the rectangular prism, and the cube.

a. Rectangular prism

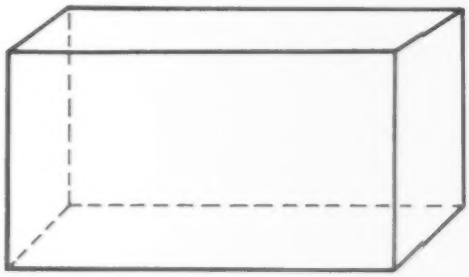
Using length l, width w, and height h, the formula for the volume V of a *rectangular prism* is:

$V = lwh$

Example: How many cubic inches of water can a fish

tank hold if its dimensions are 10 inches by 20 inches by 1 foot?

Answer: 2400 cu in.



b. Cube

A *cube* is a rectangular prism, and its volume can be determined by V = lwh. Since the length, width, and height of the cube have the same measure, namely e for edge, an alternate formula exists for the volume of a cube:

$V = e^3$

Example: If a cube has a volume of 512 cm^3, what is the length of one of its edges?

Answer: 8 cm

EVALUATION

Teachers are encouraged to develop their own formative (ongoing) evaluation strategies to monitor student achievement throughout the year. These strategies should include written and oral testing, small group work assignments, and whole class projects. Examples presented in the suggested instructional strategies section of the *Syllabus* may be used as models for examination questions or assignments.

Although there are no plans at this time to develop a State final examination for this *Syllabus*, there is discussion of developing a norm-referenced Mathematics 7-8 Test to reflect the new *Syllabus*. Such a test could be used for program evaluation and/or student placement purposes.

This *Syllabus* for mathematics in Grade 7, along with the *Syllabus* for mathematics in Grade 8, is designed to prepare students for either of two high school sequences. Those students pursuing a Regents diploma must pass Regents examinations in both Course I and Course II of the Three-Year Sequence for High School Mathematics. Those students pursuing a local diploma are required to pass either a Regents examination in mathematics, Business Mathematics, or the Regents Competency Test in Mathematics.

Students with handicapping conditions are provided with appropriate alternative testing techniques as indicated on their Individualized Education Programs (IEP). Such approved testing modifications should be used consistently in ongoing evaluations and final evaluations required for local and Regents diplomas.

APPENDICES

STATEMENT OF REGENTS GOALS FOR ELEMENTARY- AND SECONDARY-SCHOOL STUDENTS – 1984

1. Each student will master communication and computation skills as a foundation to:
   - 1.1 Think logically and creatively.
   - 1.2 Apply reasoning skills to issues and problems.
   - 1.3 Comprehend written, spoken and visual presentations in various media.
   - 1.4 Speak, listen to, read and write clearly and effectively in English.
   - 1.5 Perform basic mathematical calculations.
   - 1.6 Speak, listen to, read and write at least one language other than English.
   - 1.7 Use current and developing technologies for academic and occupational pursuits.
   - 1.8 Determine what information is needed for particular purposes and be able to acquire, organize and use that information for those purposes.

2. Each student will learn methods of inquiry and knowledge gained through the following disciplines and use the methods and knowledge in interdisciplinary applications:
   - 2.1 English language and literature.
   - 2.2 History and social science.
   - 2.3 Mathematics.
   - 2.4 Natural sciences and technology.
   - 2.5 Language and literature in at least one language other than English.

3. Each student will acquire knowledge, understanding, and appreciation of the artistic, cultural, and intellectual accomplishments of civilization and develop the skills to express personal artistic talents. Areas include:
   - 3.1 Ways to develop knowledge and appreciation of the arts.
   - 3.2 Aesthetic judgments and the ability to apply them to works of art.
   - 3.3 Ability to use cultural resources of museums, libraries, theaters, historic sites and performing arts groups.
   - 3.4 Ability to produce or perform works in at least one major art form.
   - 3.5 Materials, media and history of major art forms.
   - 3.6 Understanding of the diversity of cultural heritages.

4. Each student will acquire knowledge about political, economic and social institutions and procedures in this country and other countries. Included are:
   - 4.1 Knowledge of American political, economic and social processes and policies at national, state, and local levels.

4.2 Knowledge of political, economic, and social institutions and pro cedures in various nations; ability to compare the operation of such institutions; and understanding of the international interdependence of political, economic, social, cultural and environmental systems.

5. Each student will respect and practice basic civic values and acquire the skills, knowledge, understanding and attitudes necessary to participate in democratic self-government. Included are:

5.1 Understanding and acceptance of the values of justice, honesty, self-discipline, due process, equality and majority rule with respect for minority rights.

5.2 Respect for self, others and property as integral to a self-governing, democratic society.

5.3 Ability to apply reasoning skills and the process of democratic government to resolve societal problems and disputes.

6. Each student will develop the ability to understand and respect people of different race; sex; ability; cultural heritage; national origin; religion; and political, economic and social background, and their values, beliefs and attitudes.

7. Each student will acquire knowledge of the ecological consequences of choices in the use of the environment and natural resources.

8. Each student will develp general career skills, attitudes and work habits and make a self-assessment of career prospects. Students not directly pursuing postsecondary education will acquire entry-level employment skills.

9. Each student will learn knowledge, skills and attitudes which enable development of:

9.1 Self-esteem.

9.2 The ability to maintain physical, mental and emotional health.

9.3 Understanding of the ill effects of alcohol, tobacco and other drugs.

10. Each student will develop a commitment to lifetime learning with the capacity for undertaking new studies, synthesizing new knowledge and experience with the known, and refining the ability to judge.

STUDENTS WITH HANDICAPPING CONDITIONS

The Board of Regents, in revising the Part 100 Regulations of the Commissioner, has made a strong commitment to integrating the education of students with handicapping conditions into the total school program. According to Section 100.2(s) "Each student with a handicapping condition, as such term is defined in Section 200.2(ii) of this Chapter, shall have access to the full range of programs and services set forth in this Part to the extent that such programs and services are appropriate to such student's special educational needs." Districts must have policies and procedures in place to make sure that students with handicapping conditions have equal opportunities to access diploma credits, courses and requirements.

The majority of students with disabilities have the potential to master the curricula content requirements for a high school diploma. Most students who require special education attend regular education classes in conjunction with specialized instruction and/or related services. These students must attain the same academic standards as their nonhandicapped peers in order to meet these requirements. For this reason, it is very important that at all grade levels students with handicapping conditions receive instruction in the same content areas so as to receive the same informational base that will be required for proficiency on statewide testing programs and diploma requirements.

It is important that teachers providing instruction through this syllabus/curriculum be familiar with how the process of special education is implemented. This appendix will provide clarification as to how students with handicapping conditions participate in both regular and special education programs. The coordination and integration of both of these programs is essential for the benefit of all students.

Providing Special Education

A student who is suspected of having a handicapping condition which adversely affects the student's educational performance must be referred to the Committee on the Handicapped (COH) by either the student's parent, a professional staff member of the school, a physician, a judicial officer, the commissioner or designee of a public agency with responsibility for welfare, health or education of children, or the student, on his or her own behalf, if he or she is 18 years of age or an emancipated minor. The referral must be in writing.

The COH then must proceed to conduct an individual evaluation. This evaluation is a comprehensive assessment which is used by the COH to determine whether the student has a handicapping condition which adversely affects the student's educational performance. If the Committee on the Handicapped determines that the student is eligible for special education, the COH and the student's parents develop an individual ized education program (IEP). This recommendation must include:

- present levels of performance and individual needs according to the four criteria areas of academic, social and physical development and management needs,
- classification of the handicapping condition,
- annual goals,
- recommended program, class size, extent of participation in regular education,

- projected date for initiation of special education, amount of time per day and projected review date,
- specialized equipment and adaptive devices,
- testing modifications, and
- recommended placement

Following the COH recommendation, the Board of Education has 30 school days to arrange for implementation. After the student enters the program, a planning conference is held to complete the development of the individualized education program by adding short-term objectives and objective criteria and evaluation procedures for determining whether the instructional objectives are being met. The student's program is reviewed at least annually and the student receives a comprehensive reevaluation at least every three years.

The entire process has corresponding timelines and due process procedures for each step.

As the Committee on the Handicapped makes its recommendation for appro priate programs and placements, careful consideration is given to the concept of the least restrictive environment. The concept of least restrictive environment has three key components:

- Special education needed by the student – This means that the recommended program of a handicapped student must meet his or her individual needs. This ensures that any placement into either special or regular education programs must be appropriate. If a student is recommended for participation in a regular education setting, then the student should have abilities which would allow that student to benefit from such instruction.
- Participation with nonhandicapped peers to the maximum extent appropriate – Consideration of this component ensures the integration of handicapped students with their nonhandicapped peers, whenever appropriate to their individual needs.
- Proximity to the student's place of residence – Consideration of this component maximizes the potential interaction betwen parents and school, as well as the student's interaction with other students in the home school area.

The concept of least restrictive environment is important for all educators to understand. Its intent is to include handicapped students into the total educational system whenever possible, but it also ensures that such placements must be appropriate. Therefore, any placements that would seem to be inappropriate should constitute a referral to the COH for reconsideration.

The teacher providing instruction through this syllabus/curriculum has the opportunity to provide an educational setting which will enable the students to explore their abilities and interests. Instruction could be provided to handicapped students either by teachers certified in this subject area or by special education teachers. Teachers certified in this subject area would be providing instruction to handicapped students recommended by the COH as being able to benefit from instruction in a regular educational setting. Special education teachers may also provide this instruction to a class of students with handicapping conditions in a special class setting.

Teachers certified in the subject area should become aware of the needs of those students with handicappng conditions participating in their classes. Instructional techniques and materials must be modified to the extent appropriate to provide students with handicapping conditions the opportunity to meet diploma requirements. Information or assistance is available through special education teachers, administrators, the Committee on the Handicapped or a student's IEP.

Special education teachers providing this instruction must also become familiar with the goals and objectives of the curriculum. It is important that these teachers provide their students with the same or equivalent information contained in the curriculum.

Regardless of who provides the instruction, the cooperation between teachers of regular and special education programs is essential. It is important for the students as well as the total school environment.

Another consideration in assisting students with handicapping conditions to meet the requirements of regular education is the use of alternative testing techniques. Handicapped students may be provided alternative testing techniques on statewide tests and district testing programs. The consistent use of alternative testing techniques is important for providing handicapped students the opportunity to demon strate their knowledge and skills without being limited by their handicapping condition.

The coordination and cooperation of the total school program will assist in providing the opportunity for a greater number of handicapped students to meet the requirements needed to pursue a high school diploma. The integrated provision of regular education programs, special education programs, remediation, alternative testing techniques, modified teaching techniques and materials, and access to credit through alternatives will assist in enabling such students to pursue the high school diploma to a greater degree. The teacher who provides instruction through this curriculum has a unique opportunity to assist such students in their individual goals.

Increasing Awareness of Persons with Disabilities

Throughout the development of the Action Plan, the Board of Regents reaffirmed a strong commitment to integrating the education of students with handicapping conditions into the total school program. In keeping with the concept of total integration, the following subgoal of the Action Plan was established:

> In all subject areas, revisions in the syllabi will include materials and activities related to generic subgoals such as problem solving, reasoning skills, speaking, capacity to search for information, the use of libraries and increasing student awareness of and information about the disabled.

The purpose of this subgoal is to ensure that appropriate activities and materials are available to increase student and teacher knowledge of disabilities. Frequently students, including those with handicapping conditions, are aware of only the differences between people with and without disabilities. Teachers, whatever the subject or content, have an opportunity to assist students in gaining an awareness of the similarities of all people, including those with disabilities.

This syllabus/curriculum, by design, includes information, activities and materials regarding persons with handicapping conditions. Teachers are encouraged to include other examples as may be appropriate to their classroom or the situation at hand.

Teachers are also encouraged to assess the classroom environment to determine how the environment may contribute to student awareness of persons with disabilities. Are persons with disabilities represented in classroom displays and materials? Do books, games, posters, etc. portray children and adults with disabilities in a positive, nonstereotypical manner?

Also, teachers must be aware of the negative images inherent in some terms and avoid the use of such terms (crippled, afflicted, suffering, victim). The use of terminology which shows what people can do rather than what they can't do, and which portrays persons with disabilities accurately and positively, should be reinforced and used consistently.

It is important for students to have an opportunity for contact with people with disabilities within their daily environment. Teachers can assist by using instructional materials that include information about children and adults with disabilities. Teachers may contact the local Special Education Training and Resource Center (SETRC) for ideas and specific resources for materials that portray persons with disabilities in a positive manner.

GRADE 8: CONTENT OUTLINE AND SUGGESTED TIME ALLOTMENTS

Unit Topic — **Time (Days)**

I. An Introduction to Problem Solving — 3–4

- A. Review General Technique
- B. Problem Solving Strategies, and Extending A Problem by:
  1. Altering the problem or the question
  2. Seeking a general solution
  3. Noting restrictions to the general solution
  4. Studying cases where the general solution does not apply

II. Number Concepts — 35–38

- A. Whole Numbers
  1. Sequences and patterns
  2. Powers of whole numbers
     - a. Positive integral and zero exponents
     - b. Operations of multiplication, division, and raising to a power
- B. Integers
  1. Operations
     - a. Absolute value
     - b. Sequences with integral terms
     - c. Problem solving
  2. Properties involving operations with integers
     - a. Commutative
     - b. Associative
     - c. Distributive
     - d. Identity elements for addition and multiplication
     - e. Inverse element for addition
     - f. Closure *(optional)*
  3. Roots
     - a. Two square roots of a positive number
     - b. Principal square root
     - c. Principal cube root *(optional)*
  4. Order of operations, including roots
- C. Rational Numbers
  1. Operations and problem solving
     - a. Positive and negative common fractions

b. Positive and negative decimal fractions
2. Properties involving operations with rational numbers
3. Powers and roots of rational numbers
   a. Powers with positive integral and zero exponents
   b. Square roots that are rational numbers
4. Negative integral exponents, with natural number bases
5. Scientific notation
   a. Numbers larger than or equal to 1
   b. Numbers less than 1 but grater than 0
   c. Operations involving numbers in scientific notation: multiplication and division *(optional)*
6. Rational numbers defined

D. Irrational Numbers

1. Definition and examples
2. Rational approximation and square root
   a. Trial and error
   b. Divide and average

E. Real Numbers

1. Definition and subsets of the real numbers
2. The real number line

III. Ratio, Proportion, and Percent 22–25

A. Ratio and Proportion

1. Ratio and rate
2. Proportions and comparisons
   a. Cross-products rule and equations
   b. Order relations and proportions
3. Problem solving with proportions
   a. Consumer problems
   b. Rates, including those related to distance and time
   c. Scale drawing
   d. Similar polygons, especially triangles

B. Right Triangle Trigonometry

1. Indirect measurement
2. Naming sides in a right triangle
3. Trigonometric ratios
   a. Tangent
   b. Sine
   c. Cosine

C. Percents

1. Percents greater than 100% and less than 1%
2. Working with percents
   a. Finding a percent of a number
   b. Finding what percent one number is of another
   c. Finding a number when a percent of it is given
3. Problem solving
   a. Percent of increase and percent of decrease
   b. Compound interest

c. Applications from print media
d. Circle graphs

IV. Probability and Statistics 20–23

A. Probability

1. Review theoretical probability
2. Counting procedures related to probability
   a. Independent events
   b. Extending the counting principle
3. Permutations
   a. Definition
   b. Factorial notation
   c. n objects taken n at a time
   d. n objects taken r at a time
4. Probability of two or more events
   a. Sample space for two or more independent events
   b. Probability of two or more independent events
   c. Probability of two mutually exclusive events
   d. Applications

B. Statistics

1. Grouped data using intervals of length one
   a. The mean
   b. The median
   c. The mode
2. Presentation of organized data
   a. Histogram
   b. Frequency polygon

V. Algebra and Coordinate Geometry 35–38

A. Algebraic Expressions and Sentences

1. Forming algebraic expressions and sentences
   a. Review of terminology
   b. Translating verbal phrases
2. Evaluating algebraic expressions
3. Translating verbal sentences into formulas

B. Solving First-Degree Equations and Inequalities in One Variable

1. Properties of equality
2. A plan for solving problems algebraically
   a. General technique
   b. Equations
   c. Inequalities
3. Solutions of open sentences involving two or more steps
   a. Like terms
   b. Parentheses

C. Equations and the Coordinate Plane

1. Functions of the form $y=ax$, through applications
   a. Table of values
   b. Sentence
   c. Formula
   d. Coordinate graph
   e. Predictions
2. Functions of the form $y=ax+b$, through applications
3. Coordinate graphs of $x=a$ and $y=b$

D. Inequalities and the Coordinate Plane

1. Number line graphs of inequalities
2. Coordinate graphs of inequalities *(optional)*

VI. Computer Awareness 5–11

A. Introduction to Computers

1. Components of a modern-day digital computer *(optional)*
2. Careers *(optional)*
3. The loop as a computer process
   a. Flowcharts
   b. Programs

B. Mathematical Operations, Functions, and Relations

1. Operations using computer symbols
2. Evaluating expressions in computer format
   a. Order of operations
   b. Use of decimals in output
   c. Use of scientific notation in output
3. Functions used with computers
   a. Absolute value function, ABS (X)
   b. Greatest integer function, INT (X)
   c. Square root function, SQR (X)
   d. Random number function, RND (1)
4. Relations used with computers *(optional)*
   a. Simple inequalities
   b. Compound inequalities

C. Computer Programming *(optional)*

1. Determine computer output of given programs
2. Write original programs

VII. Geometry and Measurement 28–31

A. Geometry

1. Points, lines, and planes
   a. Points: collinear, noncollinear, coplaner
   b. Lines, segments, and rays
   c. Pairs of lines: intersecting, perpendicular, parallel, skew
2. Angles
   a. Measure of an angle

b. Angle pairs: complementary, supplementary, adjacent, vertical

3. Parallel lines cut by a transversal
4. Polygons
   a. Angle measures of polygons
      1. Sum of interior angle measures
      2. Regular polygons
      3. Exterior angles of polygons *(optional)*
   b. Triangles and quadrilaterals
5. Geometric solids
   a. Polyhedrons
1. Faces, edges, and vertices
2. Convex polyhedrons and regular polyhedrons *(optional)*
   b. Prisms
   c. Pyramids
   d. Cones
   e. Cylinders
   f. Spheres

B. Systems of Measurement

1. Metric system
   a. Review equivalent measures in metric
   b. Measures with scientific applications *(optional)*
      1. Force: newton (N)
      2. Others
2. Customary system of measurement

C. Measurement of Geometric Figures

1. Perimeter
   a. Polygons
   b. Circles
   c. Composite shapes
2. Surface area
   a. Rectangular prisms and cubes
   b. Right circular cylinders
3. Volume
   a. Prisms
   b. Right circular cylinders
   c. Pyramids
   d. Cones
   e. Spheres

D. Constructions and Activities

1. Constructions
   a. Copy a line segment
   b. Copy an angle
2. Activities involving constructions

E. The Pythagorean Theorem

1. Experiments with triangles
2. Stating the Pythagorean Theorem
3. Applications
4. Constructing irrational lengths *(optional)*

F. Congruence

1. Line segments and angles
2. Congruent figures

G. Similarity

1. Similar figures
2. Applications using proportions
3. Ratio of perimeters and ratio of areas *(optional)*